The Universality of Seesaws

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- A Plethora of Scales
- Seesaws as Dynamical Solutions
- Dialing Scales through the Universe
- Neutrinos to the Rescue?
- Concluding Remarks

A Plethora of Scales

- One of the most difficult problems to understand in particle physics is the disparity of scales.
- Two prototypical issues are the hierarchy between the scales of Gravity and the Electroweak Interactions:

$$M_P / v_F \sim 10^{17},$$

and the fine structure in the spectrum of the quarks and charged lepton masses:

$$m_q = \{5 \text{ MeV}–175 \text{ GeV}\} \; ; \; m_l = \{0.5 \text{ MeV-}2 \text{ GeV}\}$$

- Another issue is the very mass small scale associated with neutrino masses:

$$m_\nu = \{ 4\ 10^{-3}\ \mathrm{eV} - 2\ \mathrm{eV} \}$$

- Magnitude of m_ν understood from the Seesaw Mechanism [Yanagida; Gell-Mann, Ramond and Slansky]:

$$m_\nu \sim v_F^2 / M_N \text{ or } m_\nu \sim v_F^2 / M_X,$$

which relates small neutrino mass scale to much larger physical scales $\{M_N, M_X\}$ associated to right handed neutrino interactions or Grand Unification.

- Traditionally, one takes the Planck Scale related to $G_N=1/M_P^2$ [M_P^2 =1.22 10^{19} GeV]as input and asks questions about the origin of the light scales
- There is a plethora of such scales, some arising from experimental input while others are pure theoretical constructs [Table] ranging over almost 30 orders of magnitude!
- Interrelating these scales is a real challenge and requires making assumptions on physics beyond the Standard Model
- Will argue that Seesaws may provide a useful guiding principle

Scale	Physics	Value (GeV)
M_x	GUTS	$2\ 10^{16}$
M_N	RH neutrino	$10^{12}-10^{15}$
f_{PQ}	PQ breaking	$10^{9}-10^{12}$
μ_S	SUSY break	$10^{5}-10^{15}$
v_F	EW break	250
Λ_{QCD}	QCD	0.3
$\{M_H\}$	EW break	< 180
m_q	quarks	0.005-175
m_l	leptons	$5\ 10^{-4}- 2$
m_ν	neutrinos	$10^{-12}-10^{-9}$

- Only scale in Table which has a theoretically pristine origin is Λ_{QCD} , since it is set by the strong QCD dynamics: $\alpha_s(\Lambda_{QCD}^2) = 1$.
- Relation of Λ_{QCD} to M_P is logarithmic and only question is why $\alpha_s(M_P^2) \approx 1/45$ [Is this a boundary condition of Planck scale physics?]
- For QCD, because it is a dynamical theory, there is a close correlation between the physical scale Λ_{QCD} and the masses of physical states. Indeed:

$$M_{hadrons} \sim \Lambda_{QCD}$$

[m_π is an exception since $m^2_\pi \sim m_q \Lambda_{QCD}$]

- Situation much different in Electroweak Theory.

  i) it is unlikely that $v_F = [\sqrt{2}G_F]^{-1/2} \sim 250$ GeV is a dynamical scale, since precision electroweak experiments favor a light Higgs and disfavor QCD-like Technicolor Theories [$S < 0.15$]

  ii) Although m_q , m_l are proportional to v_F, the mass spectrum spanning 5 orders of magnitude suggest that the Yukawa couplings arise from physics at scales much larger than v_F

- Relation between v_F and M_P is a real problem [hierarchy problem] still poorly understood.

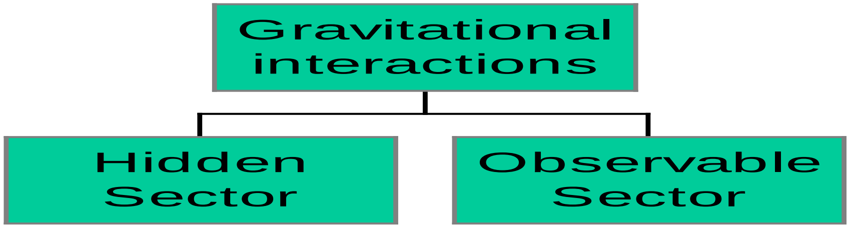
Seesaws as Dynamical Solutions

- I do not believe this problem is resolved in extra-dimensional theories, where one assumes that the Planck scale in d+4 dimensions $M^d_P = v_F$. These theories involve introducing a compactification radius R, whose scale is set by requiring that in 4-dimensions the scale of Gravity is M_P. This requires that

$$M_P \approx M^d_P (M^d_P\, R)^{d/2} = v_F (v_F\, R)^{d/2}$$

- In my view, much more satisfactory to think of v_F as originating from a Seesaw, as occurs in SUSY theories spontaneously broken in a hidden sector coupled to matter by gravity mediated interactions

- For neutrinos, via the Seesaw mechanism, one gets a small scale m_ν from a large scale M_N or M_X by relying on a known intermediate scale v_F. Thus, neutrino masses are a window on the large scale
- If nature is supersymmetric, with SUSY spontaneously broken at a scale μ_S in a hidden sector coupled to matter only gravitationally,



the superpartner masses (and other SUSY breaking parameters) are also given by a Seesaw formula

$$\tilde{m} \approx \mu_S^2 / M_P$$

- In this scenario, because of the large top Yukawa coupling, one can induce electroweak breaking from SUSY breaking.

$$\mu^2(\mu_S^2) \sim \tilde{m}^2 \rightarrow -\mu^2(v_F^2) .$$

Thus also $v_F \approx \mu_S^2 / M_P$

- If the origin of the Fermi scale is due to a SUSY induced Seesaw [$v_F \approx \mu_S^2 / M_P$], we have effectively tied this scale $v_F \sim 250$ GeV to a much larger scale $\mu_S \sim 10^{11}$ GeV
- In this Seesaw we have used known low and high scales [v_F and M_P] to infer an intermediate scale μ_S

- There might appear to be no real advantage to this, except to have shortened the gap between the Planck scale and the driving scale μ_S for the physics we observe [$M_P / \mu_S \sim 10^8$ vs $M_P / v_F \sim 10^{17}$]
- However, one can use the high scale μ_S also as the scale where family fine structure originates, with small Yukawa couplings given by Froggart-Nielsen VEV ratios: $\Gamma \sim [\langle\sigma\rangle / \mu_S]^n$, with $\langle\sigma\rangle \neq 0$ breaking an assumed family symmetry.
- In fact, one can systematically relate scales that are observable at present energies to physics at higher scales via Seesaw-like formulas [e.g. for axions, one has $m_a \sim \Lambda^2_{QCD} / f_{PQ}$]

Dialing Scales through the Universe

- These ideas run into a significant challenge when one tries to address the issue of Dark Energy in the Universe
- Einstein's equations describing the expansion of the Universe in a Robertson Walker background provide a wonderful scale-meter
- The Hubble parameter at different temperatures during the expansion provides the yardstick. Although now $H_o=(1.5 \pm 0.1)\ 10^{-33}$ eV is a tiny scale, its value varies with T as $H \sim T^2 / M_P$

- Einstein's equations

$$H^2 \equiv \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G_N \rho}{3} - \frac{k}{R^2} + \frac{\Lambda}{3}$$

$$\frac{\ddot{R}}{R} = \frac{\Lambda}{3} - \frac{4\pi G_N}{3}(\rho + 3p)$$

determine H and the Universe's acceleration once ρ, p, k, and Λ are specified.

- In a flat Universe [k=0], as predicted by inflation and confirmed observationally by WMAP, the Universe accelerates if $\Lambda > 4\pi G_N \rho_{matter}$, or, if Λ=0, a dominant component of the Universe has negative pressure and $\rho + 3p < 0$. The observed acceleration is evidence for this Dark Energy

- It is convenient to set $\Lambda=0$ and write the first Einstein equation simply as:

$$H^2 = 8\pi G_N \rho /3 + 8\pi G_N \rho_{\text{dark energy}} /3.$$

Then using an equation of state: $\omega = p/\rho$, the pure cosmological constant case, where the density is a pure vacuum energy density, corresponds to $\omega = -1$:

$$\rho_{\text{dark energy}} = -p_{\text{ dark energy}} = \rho_{\text{vacuum}} \leftrightarrow \text{constant}$$

- We know observationally that, at the present time, H_o^2 gets about 30% contribution from the first term and 70% from the second term. So we have two apparent Seesaws:

$$H_o \approx \rho_o^{1/2} / M_P \;\; ; \; H_o \approx \rho_{\text{dark energy}}^{1/2} / M_P$$

- The first Seesaw is understood in terms of known, or speculated, physics. In fact, it really is not a true Seesaw. The other Seesaw is totally mysterious!
- Because the energy density ρ depends on the Universe's scale factor R as

$$\rho \sim R^{-3(1+\omega)},$$

the contribution of $\rho_{\text{dark energy}}$ to H^2 at earlier times is negligible, so that

$$H^2 = 8\pi G_N \rho /3$$

- Since $H = H(T)$ depends on temperature, the above is really a dynamical equation, not a Seesaw. The total density just fixes the rate of expansion.

- Different components dominate ρ as the Universe expands, as they have different temperature dependences and different threshold factors.
- Schematically, one has:

$$\rho = \rho_{radiation} + \rho_{matter} + \rho_{dark\ matter}$$

with

$$\rho_{radiation} = [\pi^2/30]\, g(T) T^4$$

$$\rho_{matter} = [2\xi(3)/\pi^2]\{M_B \eta + \Sigma_i m_{\nu i}\}\, T^3$$

$$\rho_{dark\ matter} \approx \{f_{PQ}\, \Lambda_{QCD} / M_P + m^* / T^* \langle\sigma v\rangle^* M_P\}\, T^3$$

- At present [$T_o \approx 3\,°$ K] $g(T_o)=2$, so $\rho_{radiation}$ negligible while particle physics parameters {M_B, η, $m_{\nu i}$, etc} insure that ρ_{matter} and $\rho_{dark\ matter}$ contribute, respectively, 2% and 28% to H_o^2

- Situation is quite different with 2nd Seesaw. Here, if indeed one has a Cosmological Constant, so that $\rho_{\text{dark energy}} = \rho_{\text{vacuum}} = E_o^4$, one has a real Seesaw:

$$H_o \approx E_o^2 / M_P$$

which gives $E_o \approx 2\ 10^{-3}$ eV.

- What physics is associated with this very small scale? All particle physics vacuum energies are enormously bigger [e.g. $E_o^{QCD} \sim \Lambda_{QCD} \approx 1$ GeV]
- Situation is not substantially altered if $\rho_{\text{dark energy}}$ has a more dynamical origin. Although now

$$\rho_{\text{dark energy}} = \rho^o_{\text{dark energy}} [T/T_o]^{3(1+\omega)},$$

the parameters in theory difficult to understand

- An example is provided by quintessence, where one associates dark energy with a new scalar field φ which has negative pressure. One needs, in present epoch, $\rho_\varphi \approx 0.7\,\rho_c$ and $p_\varphi \approx -0.4\,\rho_c$. Hence:

$$\rho = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) \approx 0.7\left[\frac{3H_0^2}{8\pi G_N}\right] \quad ;\, p = \frac{1}{2}\dot{\varphi}^2 - V(\varphi) \approx -0.4\left[\frac{3H_0^2}{8\pi G_N}\right]$$

- The field φ is dynamical and to realize the above equations the field φ is large: $\varphi \sim G_N^{-1/2} \sim M_P$. With such large fields φ it is impossible to get the above results unless φ has nearly zero mass:

$$m_\varphi \sim E_o^2 / \varphi \;\sim H_o \approx 10^{-33}\ \text{eV}$$

- Above Seesaw is unprotected from getting big mass shifts, unless quintessence essentially decouples

Neutrinos to the Rescue?

- In a sense, the quintessence interpretation of $\rho_{\text{dark energy}}$ results in a very unpalatable seesaw:

$$m_\varphi \sim E_o^2 / M_P$$

where a difficult to understand scale $E_o \sim 2\ 10^{-3}$ eV produces, from a particle physics point of view, an even more difficult to understand scale, $m_\varphi \sim H_o \approx 10^{-33}$ eV.

- Much more satisfactory would be if one could understand $\rho_{\text{dark energy}}$ as arising dynamically from a known particle physics scale

- A very interesting suggestion along these lines has been put forward recently by Fardon, Nelson and Weiner.
- Coincidence of having in present epoch

$$\rho_{\text{dark energy}} \approx \rho^{o}_{\text{all matter}}$$

is resolved dynamically if the dark energy tracks some component of matter

- Easy to convince oneself that the best component of matter for $\rho_{\text{dark energy}}$ to track are the neutrinos
- If indeed $\rho_{\text{dark energy}}$ tracks ρ_{ν} then can perhaps also understand scale issue:

$$E_o \sim 2\ 10^{-3}\ \text{eV} \leftrightarrow m_{\nu} \sim v_F^2 / M_N$$

- Fardon, Nelson and Weiner idea is radical: neutrinos and dark energy are coupled, resulting in variable neutrino masses, which depend on neutrino density: $m_\nu = m_\nu(n_\nu)$
- In FNW picture, the energy density in the dark sector is given by (assuming, for simplicity, one neutrino flavor):

$$\rho_{dark} = m_\nu n_\nu + \rho_{dark\ energy}(m_\nu)$$

- This energy density will stabilize when

$$n_\nu + \rho'_{dark\ energy}(m_\nu) = 0$$

- The equation of state for the dark sector is readily computed:

$$\omega + 1 = -\partial \ln \rho_{dark} / 3\partial \ln R = -[R / 3\rho_{dark}]\{m_\nu \partial n_\nu / \partial R + $$
$$+ n_\nu \partial m_\nu / \partial R + \rho'_{dark\ energy} \partial m_\nu / \partial R\}$$
$$= m_\nu n_\nu / \rho_{dark} = m_\nu n_\nu / [m_\nu n_\nu + \rho_{dark\ energy}]$$

- We see that if $\omega \approx -1$ the neutrino contribution to ρ_{dark} is a small fraction of $\rho_{dark\ energy}$. Further, since we expect $\rho_{dark\ energy} \sim R^{-3(1+\omega)}$, it follows that (if ω does not change significantly with R) the neutrino mass is nearly inversely proportional to the neutrino density:

$$m_\nu \sim n_\nu^{\ \omega}$$

- I will not discuss this scenario further here, but will make just a few remarks:

i. If $\omega \approx -0.8$, the equation of state of the dark sector predicts that now

$$[m_\nu]^{cosmo} \approx 5 \text{ eV}$$

However, since $m_\nu \sim n_\nu{}^{\omega}$, if there is neutrino clustering in our galaxy the observed neutrino mass could be much smaller

$$[m_\nu]^{obs} \approx 5\,[n_\nu{}^{loc} / n_\nu{}^{cosmo}]^{\omega} \text{ eV}$$

ii. Variability of m_ν with n_ν requires reexamining many astrophysical/ cosmological issues relating to neutrinos [BBN, SN, Leptoge, ..]

iii. Although the dynamics of the dark sector is unclear, likely coupling between dark energy and neutrinos comes through SU(2) x U(1) singlet M_N:

$$m_\nu \approx v_F^2 / M_N(\varphi_{\text{dark energy}})$$

where $\varphi_{\text{dark energy}}$ is the field responsible for the dark sector dynamics

iv. The scale of the energy density associated with dark energy is of the order of that of neutrino masses, since these components of the Universe track each other, and is set by above seesaw

$$\rho_{\text{dark energy}}^{1/4} \sim m_\nu \approx v_F^2 / M_N(\varphi_{\text{dark energy}})$$

Concluding Remarks

- Hope to have shown that it is useful to imagine that the disparate scales we see in particle physics can trace their origins to some seesaw
- From this point of view, the dark energy scale $E_o \sim 2\ 10^{-3}$ eV presents a real challenge
- Speculative idea of tying the dark energy sector with the neutrino sector allows a natural seesaw explanation for E_o, but requires bold new dynamics