See-saw Mechanism and Neutrino Mass Matrix in GUT

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based on the works

in collaboration with

T.Kugo, K.Yoshioka, N. Maekawa

S.Kaneko, M.Obara, M.Tanimoto

Principle

$\boxed{\text{Naturalness}}$

no fine tuning

Seesaw Mechanism

Why is neutrino mass so small ?

Existence of Right-handed neutrinos

Small mass

can be derived

Naturally

by seesaw

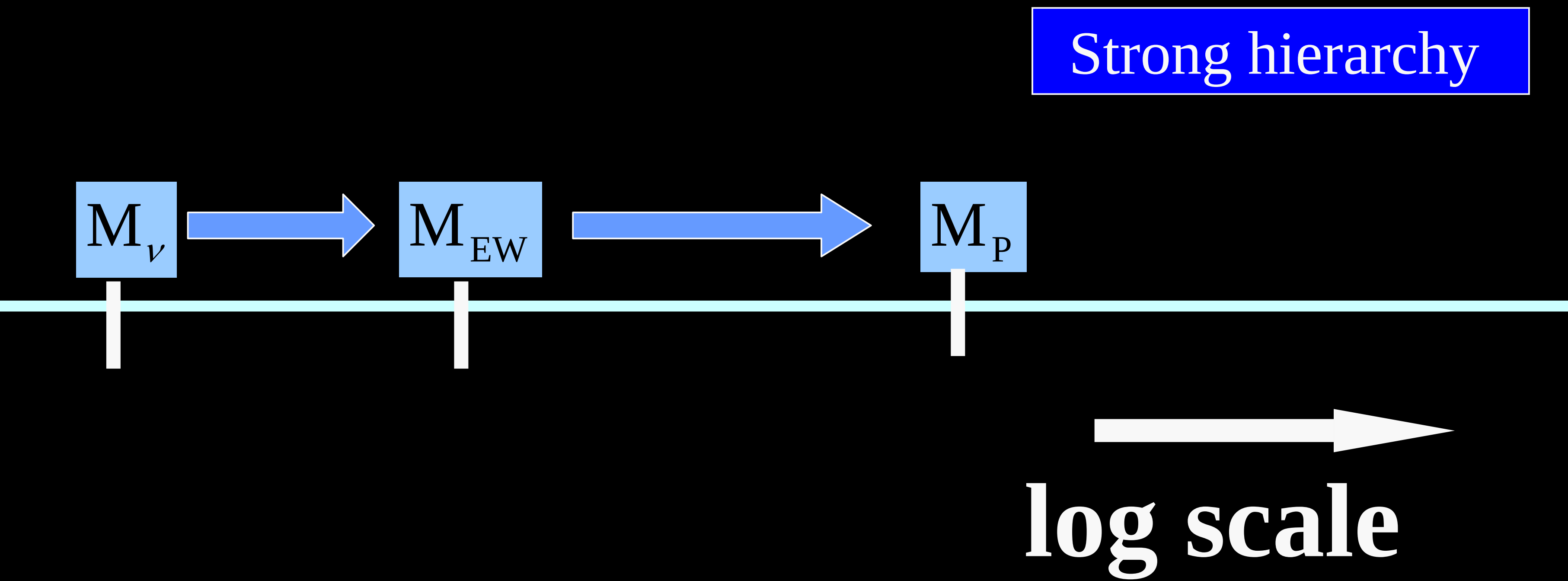
Bi-large mixing

can it be explained

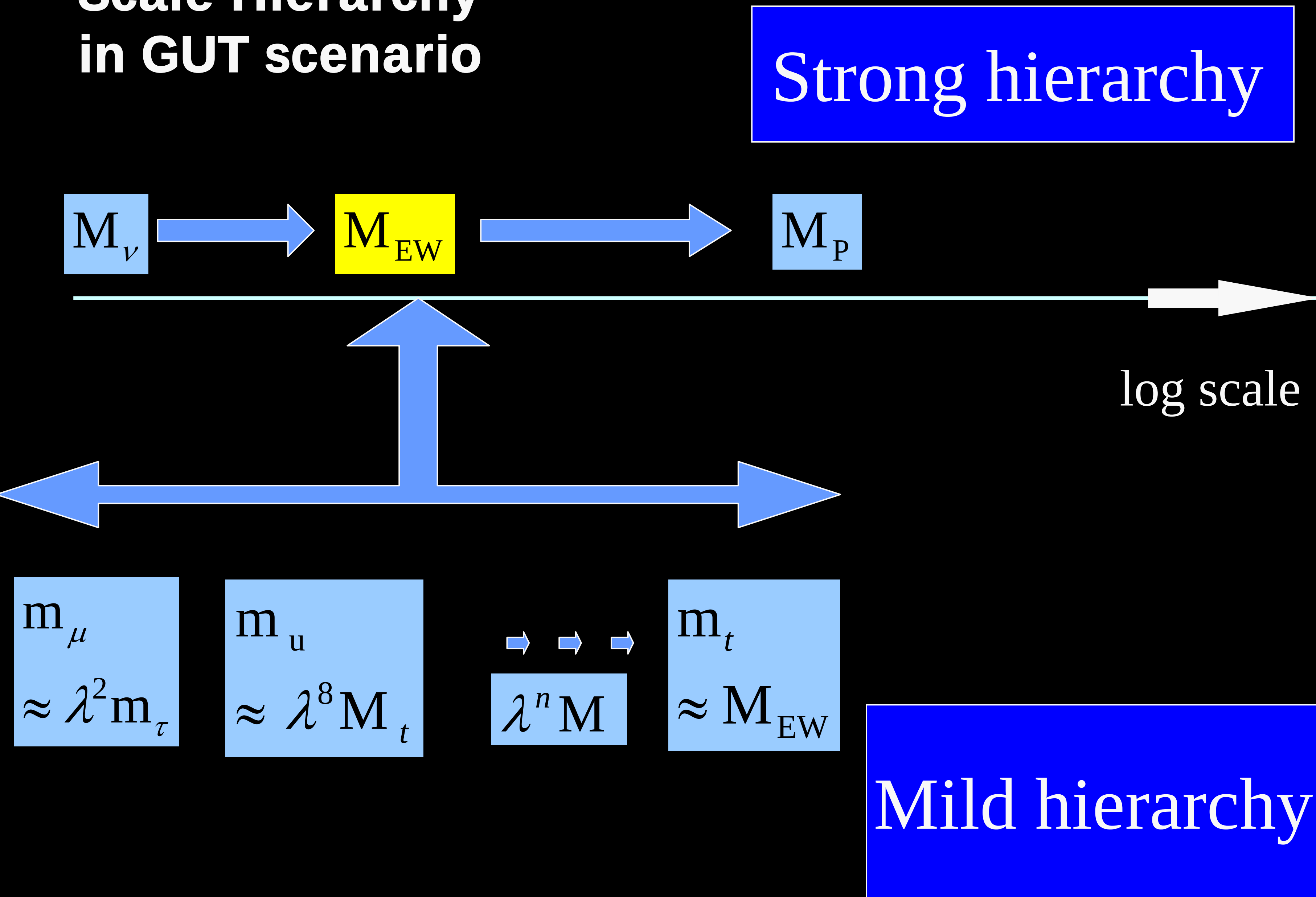
naturally ?

Hint of the origin of power Hierarchy ?

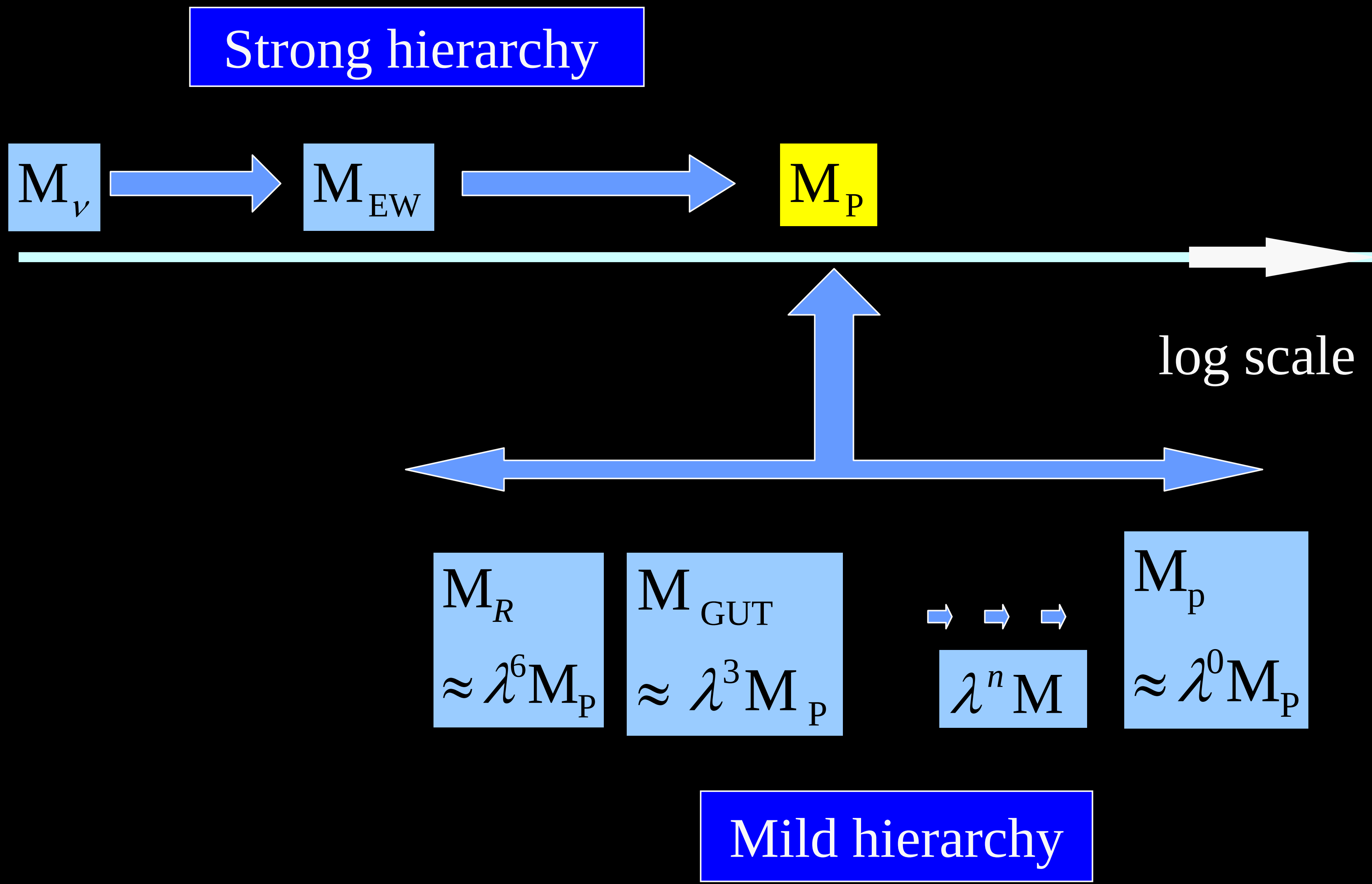
Scale Hierarchy in GUT scenario



Scale Hierarchy in GUT scenario



Scale Hierarchy in GUT scenario



Introduce …

Family Quantum number

Horizontal Symmetry
1979 Yanagida san

We use the same idea (F-N) higher dimensional Operators
Anomalous U(1) with SUSY

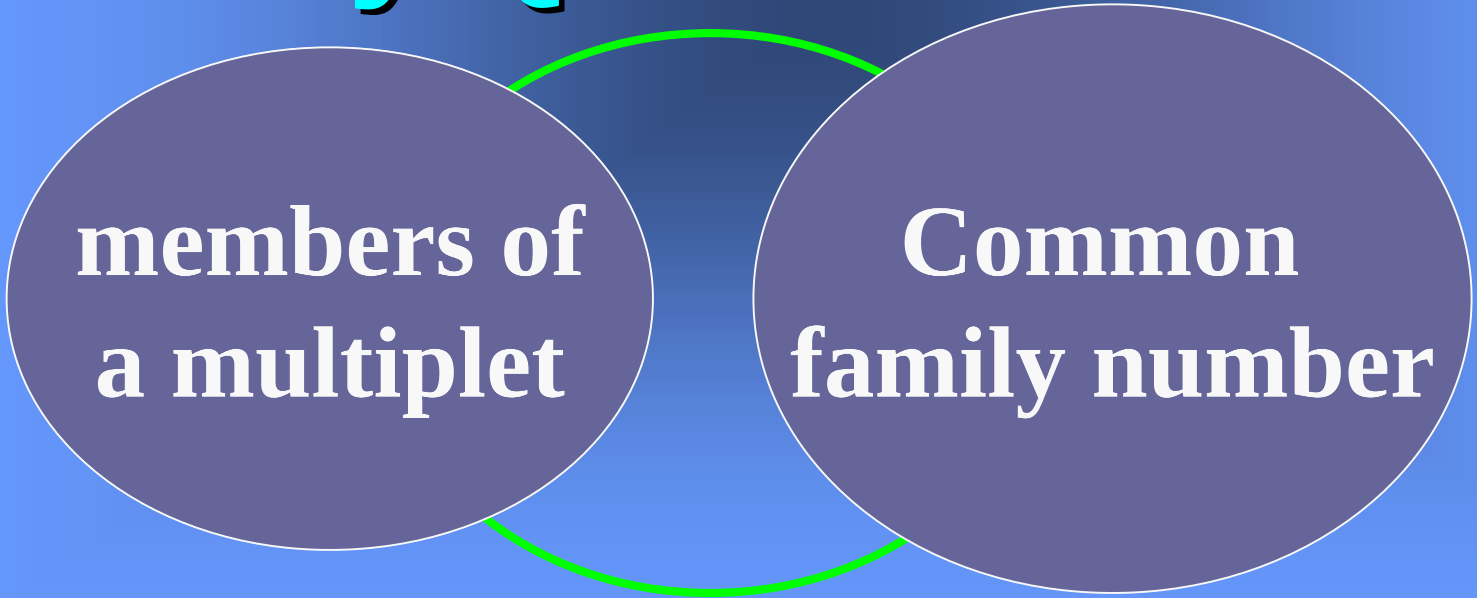
$$W = y_{ij}\ \mathrm{F_L}^{\mathrm{i}}\ \mathrm{F_R}^{\mathrm{j}}\ \mathrm{H_u}^{h} \times \left(\frac{\theta}{\Lambda}\right)^{(i+j+h)} \Rightarrow$$

$$y_{ij} \approx \mathrm{O}(1) \ \rightarrow \quad \mathrm{y}^{\mathrm{ij}}{}_{\mathrm{eff}} \equiv y_{ij}\lambda^{(i+j+h)}$$

Hierarchical parameter $\Rightarrow$ $$\lambda = \left(\frac{\langle\theta\rangle}{\Lambda}\right)^{(i+j+h)}$$

Then ...

Family Quantum number



Neutrino MNS Matrix

$$V_{MNS} = U_l^\dagger U_\nu,$$

$$\begin{aligned} U_l^\dagger M_l U_l &= \mathrm{diag}(m_e, m_\mu, m_\tau) \\ U_\nu^\dagger M_\nu U_\nu &= \mathrm{diag}(m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}) \end{aligned}$$

SeeSaw

$$M_\nu = M_{\nu_D}^T \cdot M_R^{-1} \cdot M_{\nu_D}.$$

Less hierarchical masses with Two Large Mixing Angles

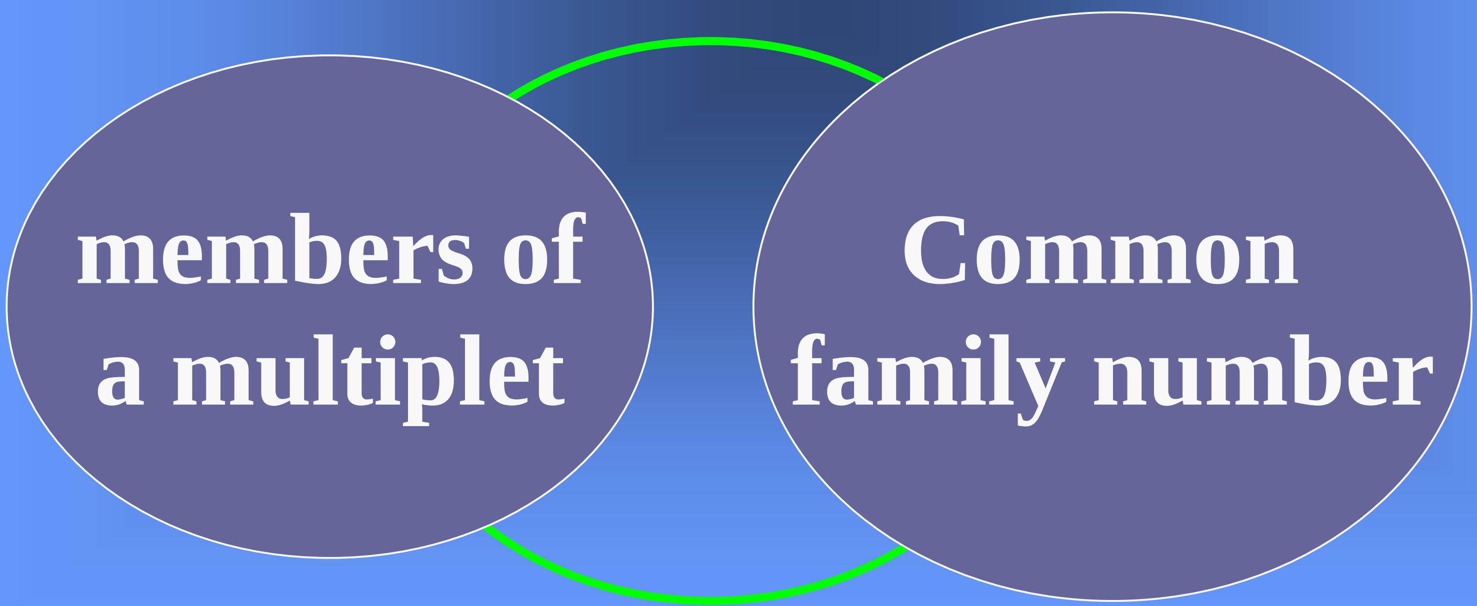
Hierarchical masses with Small mixing angles

Problem

Big difference between quarks and leptons !

Hints of family structure?

Standard model ...



23-large mixing

can it be explained

naturally ?

Minimal Matter contents

u_L

d_L

$$\begin{bmatrix} u_L \\ d_L \end{bmatrix}$$

u_R

d_R

ν_R

e_L

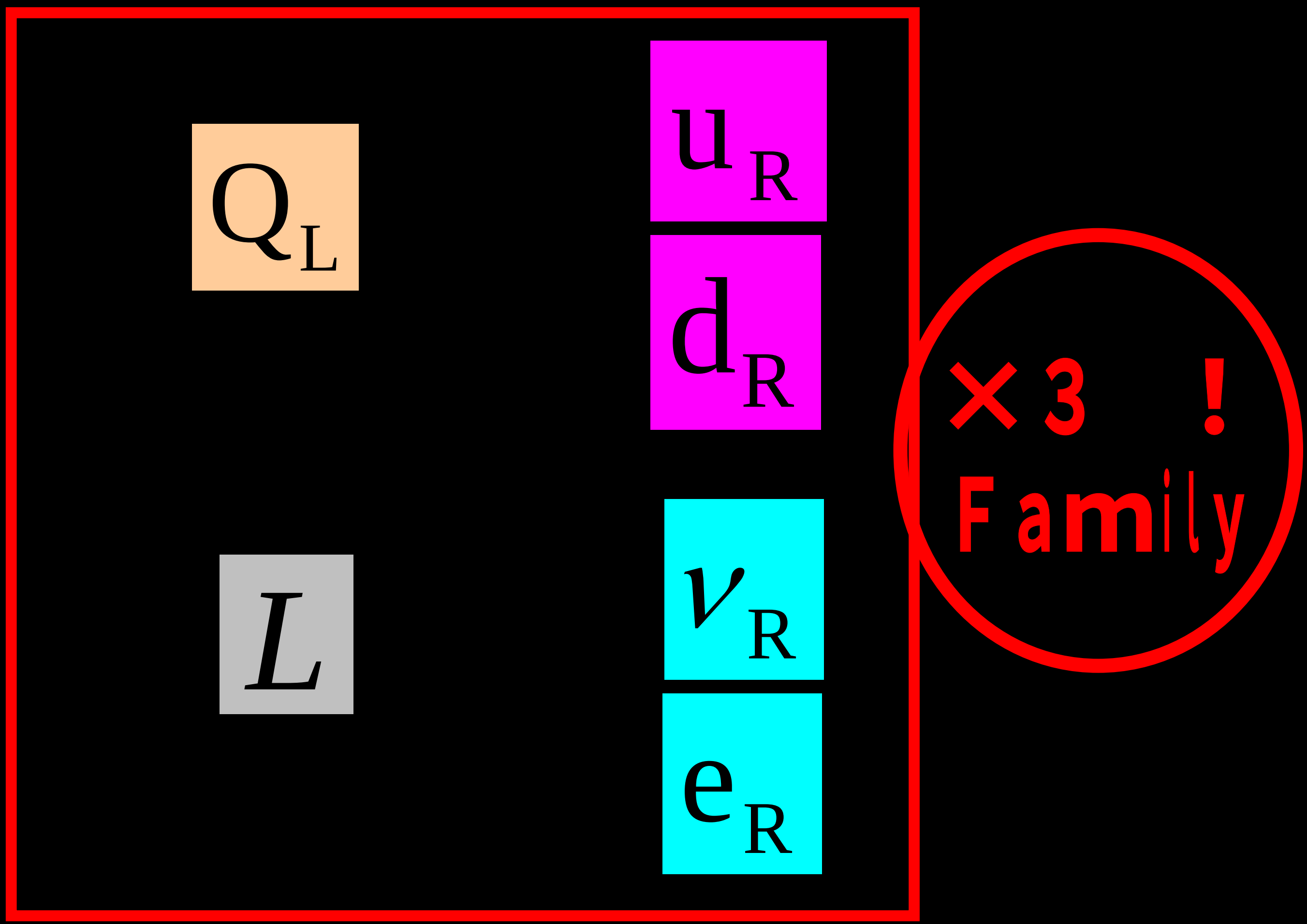
$$\begin{bmatrix} \nu_L \\ e_L \end{bmatrix}$$

ν_R

e_R

$SU(2)_L$

Standard Symmetry



Simplest Scenario with Family Symmetry

based on the works

in collaboration with

T. Kugo

$$M_u \approx m_t \begin{pmatrix} \lambda^{6-7} & * & * \\ * & \lambda^4 & * \\ * & * & 1 \end{pmatrix}$$

Quark

Masses

and

mixings

$\boldsymbol{\lambda = \lambda_c}$

$$M_d \approx m_t \begin{pmatrix} \lambda^{6} & * & * \\ * & \lambda^4 & * \\ * & * & \lambda^2 \end{pmatrix}$$

$$U_{CKM} \approx \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

Up Quark Masses

$$\frac{m_u}{m_c} \approx \lambda^4$$

$$\frac{m_u}{m_t} \approx \lambda^{6-7}$$

If we introduce

$$X(u_R, c_R, t_R) = (3-4, 2, 0)$$

$$\begin{array}{cc} & \begin{bmatrix} 3-4 & 2 & 0 \end{bmatrix} u_R \\ Q_L & \end{array}$$

$$M_u \approx \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} \begin{pmatrix} \lambda^{6-7} & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} m_t$$

$$\frac{m_d}{m_b} \approx \lambda^4$$

$$\frac{m_s}{m_b} \approx \lambda^2$$

$$\frac{m_b}{m_t} \approx \lambda^2$$

Down Quark Masses

If we introduce

$$X(u_R, c_R, t_R) = (3,2,2)$$

$$\begin{array}{ccc} & Q_L & \begin{bmatrix} 3 & 2 & 2 \end{bmatrix} d_R \\ M_d \approx & \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} & \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & 1 & 1 \end{pmatrix} \lambda^2 m_t \end{array}$$

Standard symmetry

No relation between quarks and leptons

Simplest Scenario
with Family Symmetry

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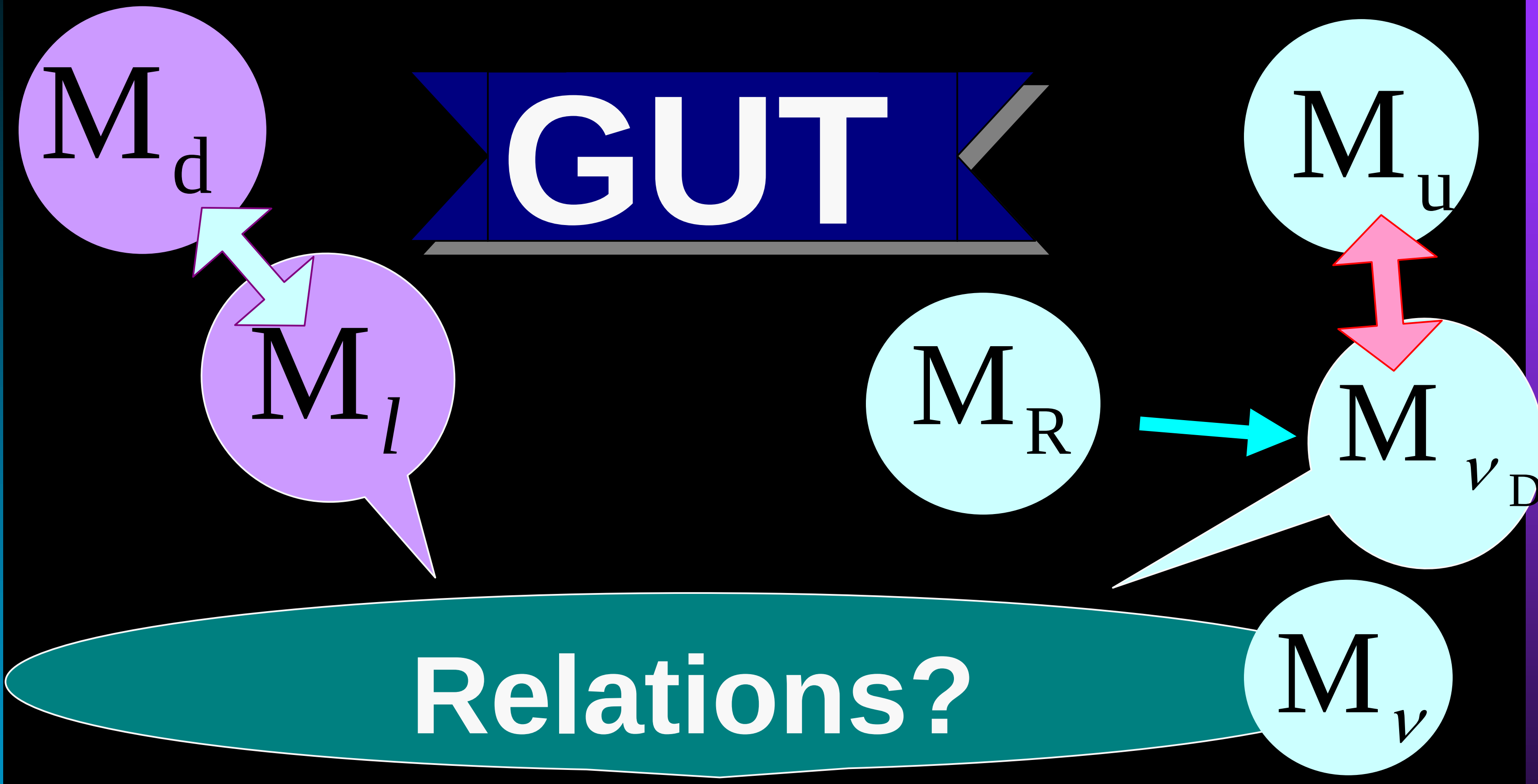
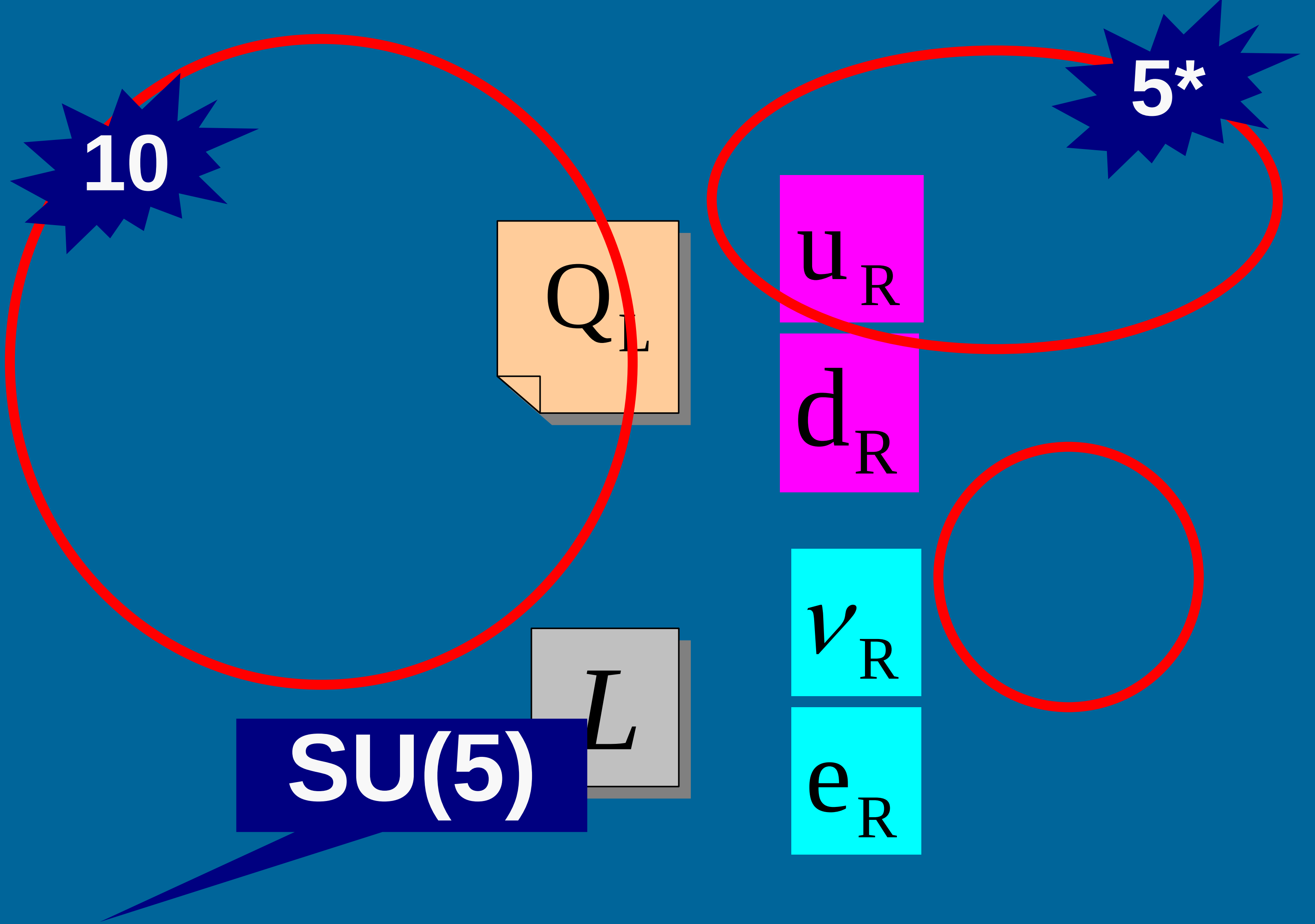
GUT

with

Family Quantum number


GUT
M_d
M_l
M_u
M_R
M_{ν_D}
M_ν
Relations?



10
5*
Q_L
u_R
d_R
ν_R
e_R
L
SU(5)

Quark Masses mixings

It is automatical

$$X(10_1, 10_2, 10_3) = (3,2,0)$$

$$U_{u_L} \approx U_{d_L} \approx \left[\lambda^{|10_i - 10_j|} \right]$$

$$U_{CKM} \approx \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

Down Quark
charged leptons

$$X(5^*{}_1,5^*{}_2,5^*{}_3)=(3,2,2)$$

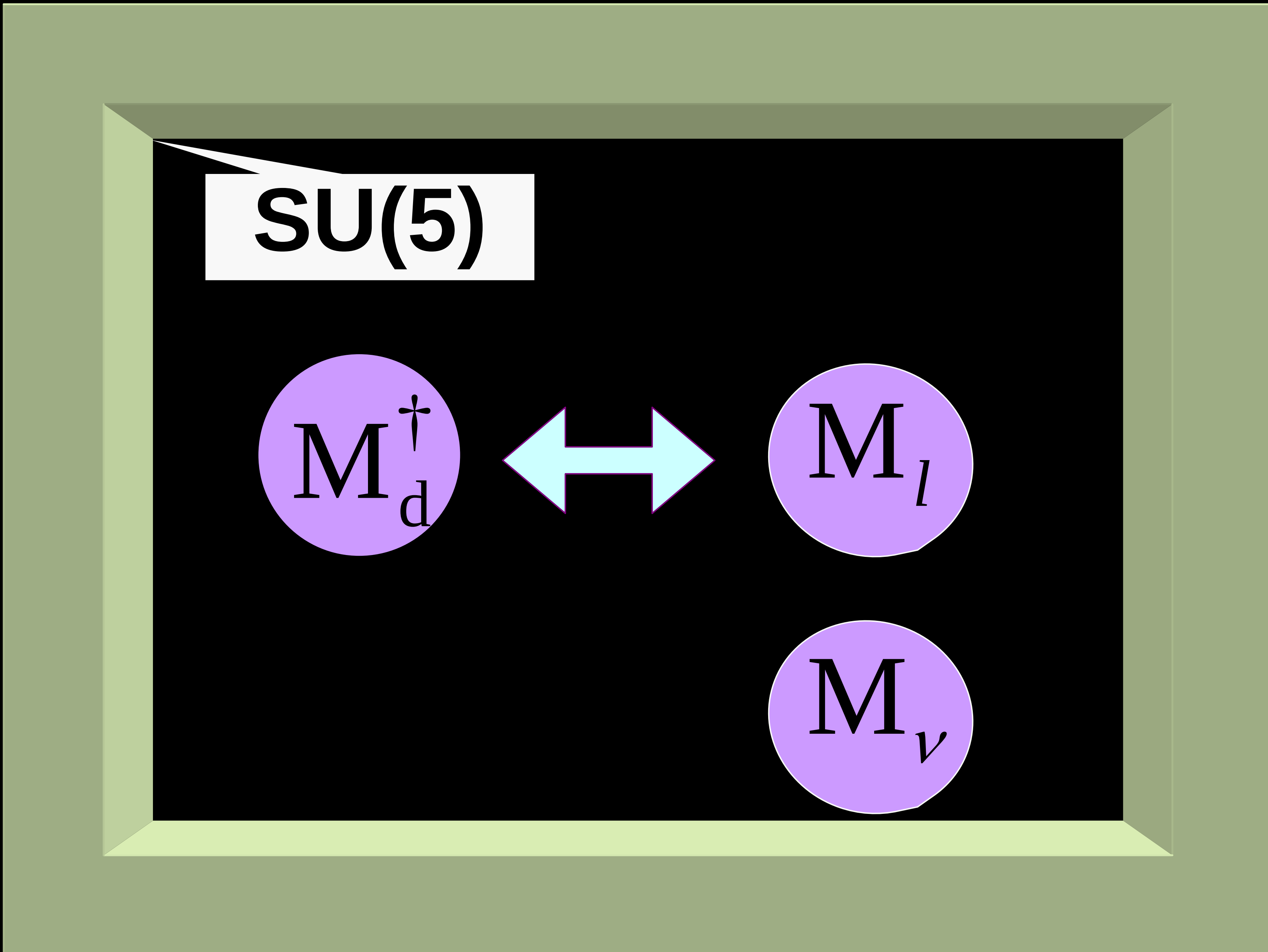
$$M_{\mathrm{d}} \approx \begin{array}{cc} Q_L & \begin{bmatrix}3 & 2 & 2\end{bmatrix}\ d_{\mathrm{R}} \\ \begin{bmatrix}3\\2\\0\end{bmatrix} & \begin{pmatrix}\lambda^4 & \lambda^3 & \lambda^3\\ \lambda^3 & \lambda^2 & \lambda^2\\ \lambda & 1 & 1\end{pmatrix}\lambda^2 m_t \end{array}$$

$$\Rightarrow M^{\dagger}{}_l \approx \begin{pmatrix}\lambda^4 & \lambda^3 & \lambda^3\\ \lambda^3 & \lambda^2 & \lambda^2\\ \lambda & 1 & 1\end{pmatrix}\lambda^2 m_t$$

Neutrino mass matrix

$$X(\nu_{L1},\nu_{L2},\nu_{L3})=(3,2,2)$$

$$M_{\nu} \approx (\lambda^{\left|5^*{}_i+5^*{}_j\right|}) \approx \begin{bmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{bmatrix}$$



SU(5)
$M_d^\dagger$
M_l
M_ν

Thus simple U (1) FN uniquely dictates the forms:

$$M_\nu \propto \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}$$

Also
We have

$$U_l \approx m_b \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}$$

Possibly, (Maekawa version)

N.Maekawa, PTP 106(2001)401, and others

$$M_d \approx \begin{pmatrix} \lambda^4 & \lambda^3\sqrt{\lambda} & \lambda^3 \\ \lambda^3 & \lambda^2\sqrt{\lambda} & \lambda^2 \\ \lambda & \sqrt{\lambda} & 1 \end{pmatrix} \lambda^2 m_t$$

$$M_\nu \approx \begin{pmatrix} \lambda^2 & \lambda\sqrt{\lambda} & \lambda \\ \lambda\sqrt{\lambda} & \lambda & \sqrt{\lambda} \\ \lambda & \sqrt{\lambda} & 1 \end{pmatrix} \lambda^{?} m_3$$

Up to here 2-3 family structure almost determined !

Question 1

Question 1

Two large mixing angles？

Mass ratio ？

1 Order of magnitude
does not work !

2 naturally reproduced?

This sub-matrix should have Its Det of order of λ ? Non-trivial!

However note that once it is realized.....

1-2large mixing is automatically realized!

$$M_\nu \approx \begin{bmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{bmatrix}$$

Maekawa version (E6)

This submatrix
Det = λ
Naturally we have order of λ !

$$M_\nu \approx \begin{pmatrix} \lambda^2 & \lambda\sqrt{\lambda} & \lambda \\ \lambda\sqrt{\lambda} & \lambda & \sqrt{\lambda} \\ \lambda & \sqrt{\lambda} & 1 \end{pmatrix} \lambda^{?} m_3$$

$$V_{MNS} = U_l^{\dagger} U_\nu$$

M_l

M_ν

either

or

Without fine tuning

Which reproduces large mixing angle?

Table 1 Four CHOOZ limit igin of large mixing

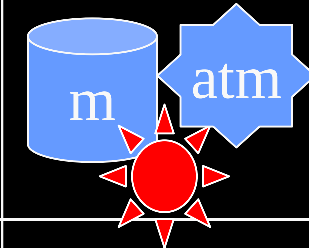
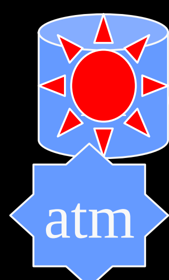
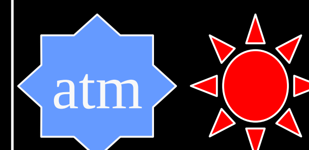
	case A	~~case B~~	case C	case D
solar	M_l	M_l	M_ν	M_ν
atom	M_l	M_ν	M_l	M_ν

Down-road

Up-Down

Up-road

Provable texture

OK	Simple U(1)FN			
M_ν	$\begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}$	$\begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}$	$\begin{pmatrix} \lambda & \lambda & \\ \lambda & \lambda & \\ & & 1 \end{pmatrix}$	$\begin{pmatrix} \lambda^{2\leq} & & \\ & \lambda & \\ & & 1 \end{pmatrix}$
$M^T{}_l M_l$	$\begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}$	$\begin{pmatrix} \lambda^8 & & \\ & \lambda^4 & \\ & & 1 \end{pmatrix}$	$\begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$
Need Little tuning				

Can we reproduce the neutrino large mixing out of hierarchical Dirac Masses?

Impossible !!! Unless fine tuning

Needs Zero structure to relate Hierarchical Up quark mass matrix for Neutrino Dirac Masses

Parallel Family Structure Can survive !!!!

$$\mathrm{M}_\nu = m^T M^{-1} m$$

$$If \quad (m)_{ij} \propto \lambda^i \lambda^j$$

then

$$\mathrm{M}_\nu \propto m$$

M.B,S.Kaneko, M.Obara and M.Tanimoto
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In order to get such desirable neutrino mass matrix

Needs Zero structure to relate Hierarchical Up quark mass matrix to Neutrino Dirac Masses

→ seesaw enhancement can occur

Tanimoto san

Question 2

Question 2

Origin of difference between 5*and 10 ?

1 Higher dimension or higher GUT ?

2 naturally reproduced?

$Q_L, u_R\ e_R$ d_R, L ν_R 1st

$Q_L, u_R\ e_R$ d_R, L ν_R 2nd

$Q_L, u_R\ e_R$ d_R, L ν_R 3rd

GUT ⇔ charge quantization
gauge unification
anomaly free set

27

$Q_L, u_R e_R$ $d_{R,L}$ ν_R 1st 3

$Q_L, u_R e_R$ $d_{R,L}$ ν_R 2nd 2

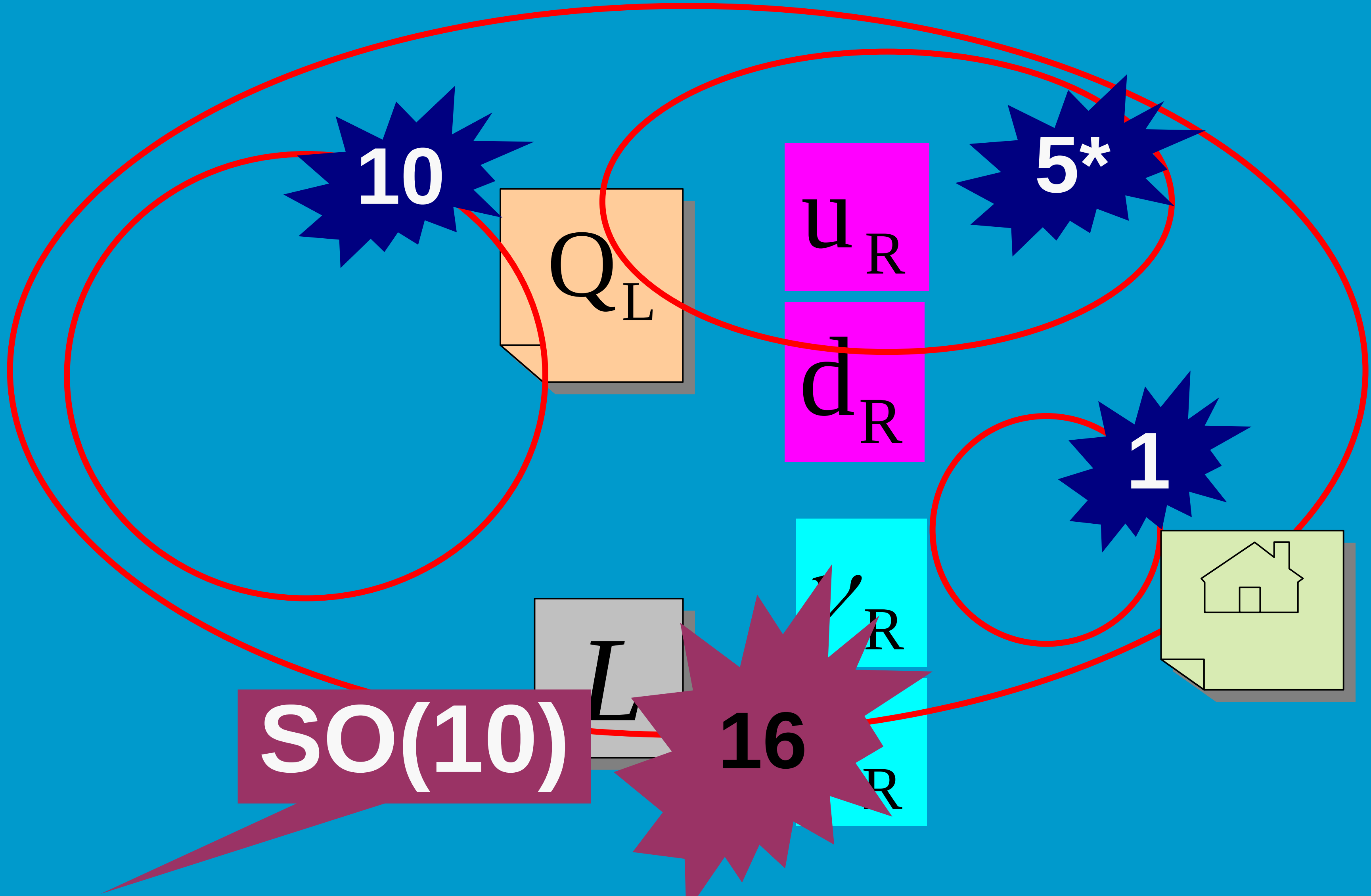
$Q_L, u_R e_R$ $d_{R,L}$ ν_R 3rd 0

$d_{R,L}$ ν_R

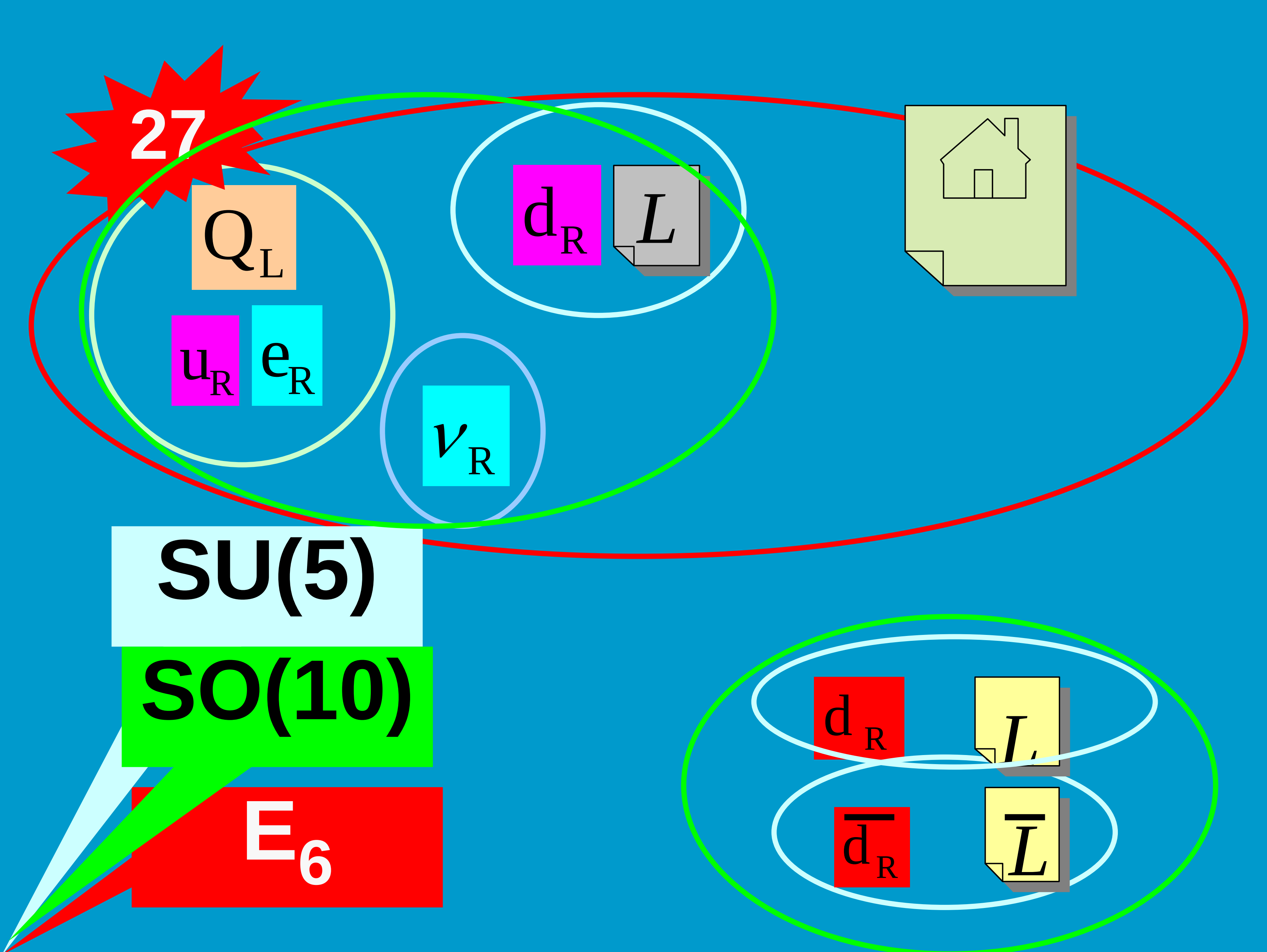
$d_{R,L}$ ν_R

$d_{R,L}$ ν_R

Doubling ⇔ same family number



10
5*
1
Q_L
u_R
d_R
ν_R
L
e_R
16
SO(10)


27
Q_L
u_R
e_R
d_R
L
ν_R
SU(5)
SO(10)
E_6
d_R
L
$\bar{d}_R$
$\bar{L}$

E_6

Natural scenario
to reproduce
Family twisting

Non Parallel Family Structure

not a mere repetition

We need some new idea for family structure

Interesting to find the reason

Why the nature choose
the world where

10s are governed by King
5*s live in democratic society

Anarchy model, Brane world....

Another Interesting fact is

Even if we start hierarchical Dirac mass, we can reproduce bi-large mixing.

SO(10) with antisymmetric Yukawa, zero structure….

Nature chooses?

Family twisting structure

Or ?

Parallel Family structure

That is a question!

My History on Neutrino

Pre-history
Yanagida san at Yukawa Indstitute

- SI1995 post YKIS The first SI in Japan
  Smirnov Lecture
- SI1997 SI1
  Yanagida san talk family twisting structure
- SI1998 pre-SI
  Takayama conf.
  Ramond san at Aspen Center

seesaw

Atm neutrino data

Solar Neutrino data

Good

1979	Seesaw
1987	Super Nova data
1996	SK data
1997	Takayama C
1998	
1999	K2K
2000	
2001	Solar Neutrino LMA
2002	KamLand SNO
2003	WMAP
2004	Oscillation L/E dep.

Yanagida san at Yukawa Institute

Only two physicists
Were at the seminor

Yanagida san at SI 1997
SK preliminary data

Atm Neutrino
Mass and mixing

Ramond san
at Aspen center

F-N family number
Works well !

We should make
Full use of E6 GUT

Now we know・・・・・

Mixing angles

Almost maximal **large**

$$\sin^2 2\theta_{atm} \qquad \tan^2 \theta_{sol}$$

Mass Differences

$$\Delta m_{\mathrm{sol}}^2 \Big/ \Delta m_{\mathrm{atm}}^2 \approx \lambda \qquad \cancel{\lambda^2}$$

Kugo san can manage exceptional group !

F-N family +
Family twisting structure

We can make
Full use of E6 GUT