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M-Theory Phenomenology and See-Saw Mechanism

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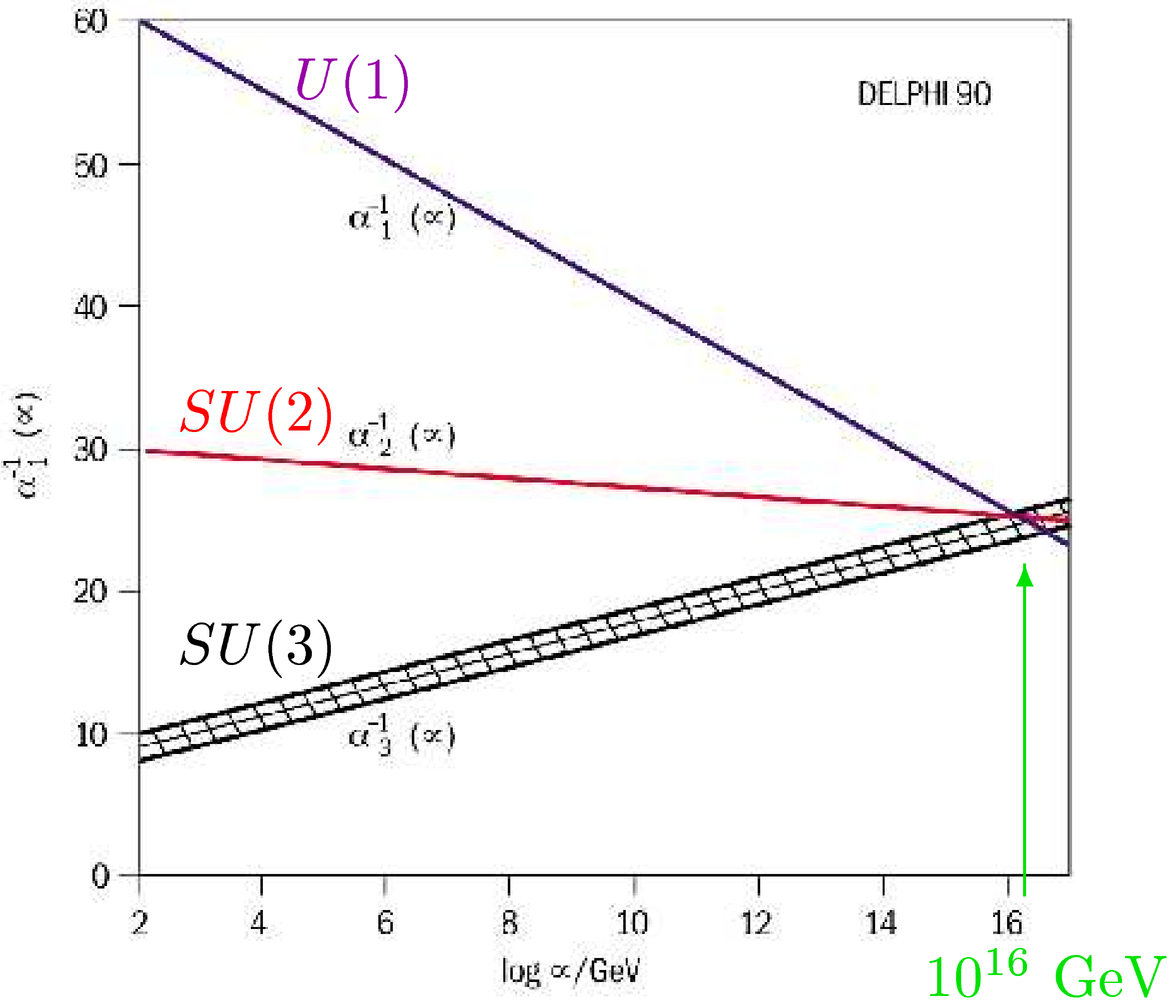
Contents:

1 Introduction

Two significant facts which imply the necessity of going to ultra-high energy.

(1) Renormalization Group



(2) See-Saw Mechanism

$$\frac{m^2}{M} \sim \frac{(100\ \mathrm{GeV})^2}{10^{16}\ \mathrm{GeV}} = 10^{-12}\ \mathrm{GeV} = 1\ \mathrm{meV}$$

These do not necessarily imply the Planck mass but they are very close enough.

$\longrightarrow$ *String Theory*

- Old generation (P. Ramond) worked both in string theory and phenomenology.
- New generation (except for a few examples) works either in string or phenomenology.

$\longrightarrow$ *Unhealthy Condition!*

Here I would like to start from M-theory and try to go all the way to calculate the quark-lepton mass matrices, mixing angles and so on. (ref. E. Witten, hep-ph/02021018)

2 M-theory

M-theory $\xrightarrow{\text{low energy}}$ 11-dimensional supergravity

- Matrix formulation
- Membrane formulation
  - effective $\rightarrow$ D-brane
  - fundamental $\rightarrow$ (?)
- It looks non-renormalizable (?)
- It lacks small expansion parameter

To investigate the possibility of fundamental membrane theory is a worthwhile thing to do.

Closed membrane

$$\langle 0|S|0\rangle = \sum_{\substack{\text{all closed} \\ \text{3-manifolds}}} \int e^{-\int \mathcal{L} d^3\sigma}$$

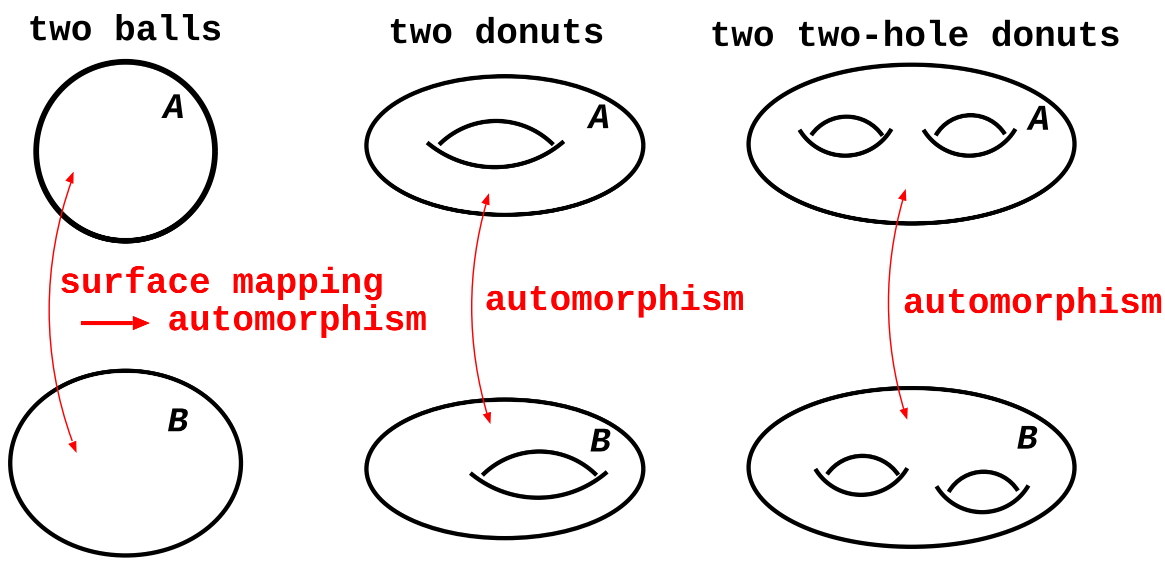
3-manifold

(i) Topology (ref. H. S., hep-th/0304164)

(ii) Geometry g_{ij}

(i) Topology

Heegaard diagram: any 3-manifold can be expressed in one or more than one Heegaard diagram.



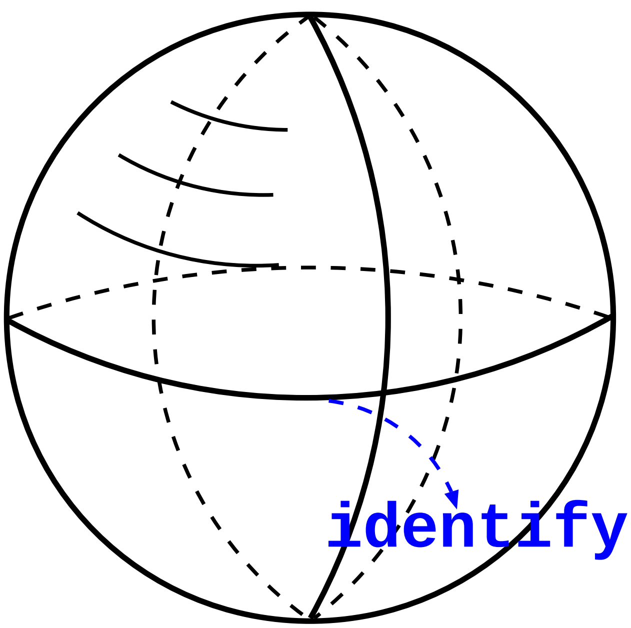
classification of surface automorphism

$\Longleftrightarrow$ classification of topology

♢ Two-balls

$$S^3: \; x^2 + y^2 + z^2 = 1$$
$$(\mathrm{A}): \; x^2 + y^2 = 1 - z^2 \le 1, \;\; z \ge 0$$
$$(\mathrm{B}): \; x^2 + y^2 = 1 - z^2 \le 1, \;\; z \le 0$$

two donuts $\rightleftharpoons$ lens space $L(p,\, q)$



p-sections in upper and and lower hemisphere $\rightarrow \dfrac{2\pi}{p} \times q$ rotation and identify upper and corresponding lower section

(i) Geometry

Thurston conjecture:

> *"Any geometry can be brough into the following 8 geometries by a diffeo(homeo)morphism:*
> H^3, S^3, E^3, $S^2 \times S^1$, $H^2 \times S^1$,
> Sol, , Nil, *and* $SL(2, R)$ *"*

No need to integrate over g_{ij}. This situation is similar to the string case where g_{ij}-integration is reduced to a finite number of moduli integration for a given topology.
$\Longrightarrow$ suggestive of renormalizability

Unitarity

1. light-cone gauge ?
2. dual string theory ?

3 Low-energy effective action

11-dimensional supergravity

"vacuum degeneracy" $\longleftrightarrow$ supersymmetry

This implies that we should look for a non-generic M-theory which admits only restricted $g_{\mu\nu}$ or $A_{\mu\nu\rho\kappa}$ leading to the unique vacuum (mirror symmetry (?)).

Question: Does the phenomenologically correct vacuum exist among the possible vacua ? Then, we can work backwards to find a principle to get this vacuum.

Witten:

$$11 = \underbrace{4}_{\text{Minkowski}} + \underbrace{7}_{\substack{\text{compactified manifold} \\ \text{of } G_2 \text{ holonomy}}}$$

$$7 = \underbrace{4}_{K_3 \text{ manifold}} +3$$

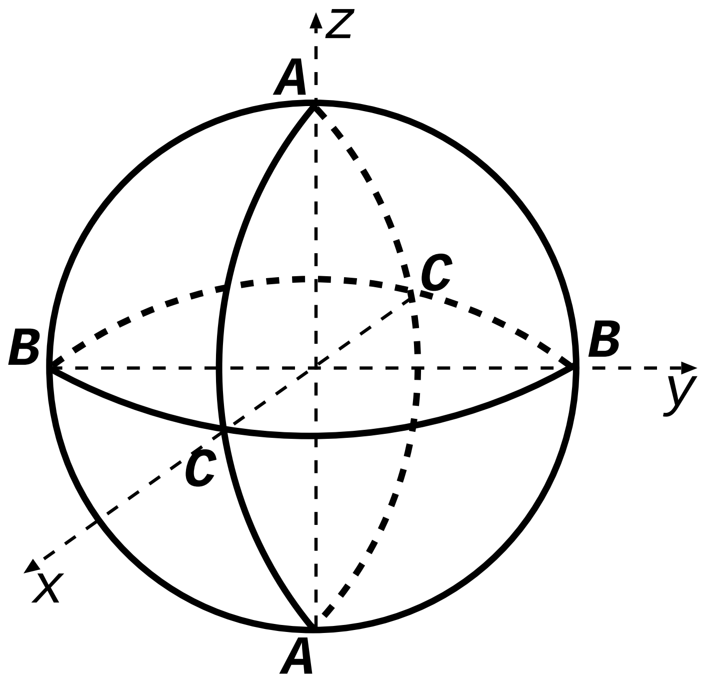
4 space (K_3 manifold) has a singularity (monodoromy) to produce a given gauge symmetry. This gauge symmetry is localized in the 3 space.

"The supersymmetry may also be localized in 3 space." H. S.

Assumption about the vacuum

1. 4 space (K_3 manifold) gives $SO(10)$ group symmetry
2. 3 space is a lens space $L(4,2)$ with certain discrete symmetry

♠ Lens space $L(4,2)$



Discrete symmetry

$$\begin{cases} (1) & y \to -y \\ & y = 0, \text{ surface, } B \\ (2) & z \to -z \\ (3) & x \to -x \end{cases}$$

A, B, C, O are the only fixed points. We assume that the theory is invariant under these transformations and the symmetry is localized in these points.

$$A, B, C \to SO(10), \quad O \to U(1) \times U(1)$$

4 Explicit model

Chiral multiplet:

$Q^{(i)}_{16}$ $(i = 1, 2, 3)$ $1 \leftrightarrow A$, $2 \leftrightarrow B$, $3 \leftrightarrow C$

$H^{(1)}_{10}$ only at A $\oplus H^{(i)}_{45}$ $(i = 1, 2, 3)$

call this the *physical chiral sector.*

Gauge fields:

$$V^{(i)}_{45} \quad (i = 1, 2, 3)$$

"Hidden" sector (point O) $U_y(1) \times U_z(1)$:

$H(y_i)$, $H(-y_i)$ $(i = 1, 2, 3)$

$H(z)$, $H(-z)$

Messengers:

$Q^{(i)}_{16}(y_i)$, $Q^{(i)}_{\overline{16}}(-y_i)$ $(i = 1, 2, 3)$

$H^{(1)}_{10}(z)$, $H^{(1)}_{10}(-z)$

Discrete symmetry

$x,\ y$ reflection trivial

z reflection

$$
\begin{array}{rcll}
Q_{16}^{(i)} & \to & e^{i\alpha_{16}^{(i)}} Q_{16}^{(i)} & \alpha_{16}^{(i)} = -\pi/4 \\
H_{10}^{(1)} & \to & e^{i\alpha_{10}} H_{10}^{(1)} & \alpha_{10} = \pi/2 \\
H_{45}^{(i)} & \to & H_{45}^{(i)} &
\end{array}
$$

$$
\begin{array}{rcl}
H(y_i) & \to & H(y_i) \\
H(-y_i) & \to & H(-y_i) \\
H(z) & \to & H(z) \\
H(-z) & \to & H(-z)
\end{array}
$$

$$
\begin{array}{rcll}
Q_{\frac{(i)}{16}}(\mp y_i) & \to & e^{i\alpha_{\frac{(i)}{16}}(\mp y_i)} Q_{\frac{(i)}{16}}(\mp y_i) & \alpha_{\frac{(i)}{16}} = \pm\pi/4 \\
H_{10}(z) & \to & e^{i\alpha_{10}(z)} H_{10}(z) & \alpha_{10}(z) = -\pi/2 \\
H_{10}(-z) & \to & e^{i\alpha_{10}(-z)} H_{10}(-z) &
\end{array}
$$

$$\alpha_{10}(-z) = 0 \ \text{ or } \ \pi/2$$

Superpotential

$$
\begin{aligned}
W = & \, g Q_{16}^{(1)} Q_{16}^{(1)} H_{10}^{(1)} \\
& + g_i Q_{16}^{(i)} H(y_i) Q_{\overline{16}}^{(i)}(-y_i) + h H_{10}^{(1)} H(-z) H_{10}^{(1)}(z) \\
& + f_i Q_{16}^{(i)}(y_i) H_{45}^{(i)} Q_{\overline{16}}^{(i)}(-y_i) + f_{10} H_{10}^{(1)}(z) H_{45}^{(1)} H_{10}^{(1)}(-z) \\
& + m_i Q_{16}^{(i)}(y_i) Q_{\overline{16}}^{(i)}(-y_i) + m_{10} H_{10}^{(1)}(z) H_{10}^{(1)}(-z) \\
& + M_i H(y_i) H(-y_i) + M_0 H(z) H(-z)
\end{aligned}
$$

(Note)

1. Second and third generations do not couple to Higgs directly.
2. No mass term to $H_{10}^{(1)}$ or $H_{10}^{(1)}(\pm z)$.
3. $H_{45}^{(i)}$ couples only to $Q_{16}^{(i)}(y_i)$, $Q_{16}^{(i)}(-y_i)$ and $H_{10}(z)$, $H_{10}(-z)$.
4. The discrete symmetry will not be used for the explanation of doublet-triplet splitting.
5. Coupling constants can be calculated in principle in terms of membrane instanton. Hierarchical values are expected.

5 Broken symmetry and hierarchy

- Supersymmetry breaking
- $\{SO(10)\}^3 \to SU(3) \times SU(2) \times U(1)$

Supersymmetry breaking

D-term for U_z and U_y will violate supersymmetry. This is to avoid the non-renormalization theorem which results in

$$\langle Q_{16}^{(i)}(y_i) Q_{\overline{16}}^{(i)}(-y_i)\rangle \propto \delta_{ij} \;\Rightarrow \{SO(10)\}^3 \to SO(10)$$

$$\langle H_{10}(z) H_{10}(-z)\rangle = 0 \quad \Rightarrow \text{Higgs mass}$$

even in the non-perturbative calculation.

Explicitly, for $\kappa_y D_y + \kappa_z D_z$ and

$$\frac{|M_i|^2}{g_y y_i} < \kappa_y < \frac{|m_i|^2}{g_y y_i}, \; \frac{|M_0|^2}{g_z z} < \kappa_z < \frac{|m_{10}|^2}{g_z z}$$

$$\langle H(z)\rangle_F = M_0 \langle H(-z)\rangle_A \neq 0$$

$$\langle H(y_i)\rangle_F = M_i \langle H(-y_i)\rangle_A \neq 0$$

Consequences

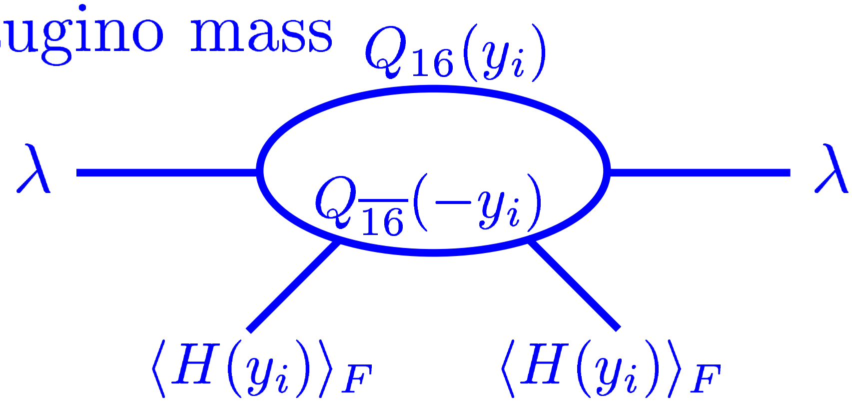
$$g_i Q_{16}^{(i)}(y_i)\langle H(y_i)\rangle_F Q_{\overline{16}}^{(i)}(-y_i)$$

$$\Longrightarrow \text{squark mass } \tilde{g}_i \frac{\langle H(y_i)\rangle_F^2}{m_i} \text{ see-saw}$$

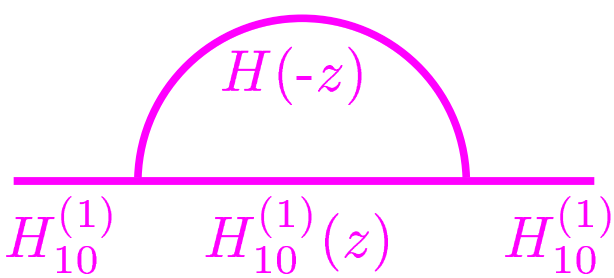
$$hH_{10}^{(1)}\langle H(-z)\rangle_A H_{10}^{(1)}(z)$$

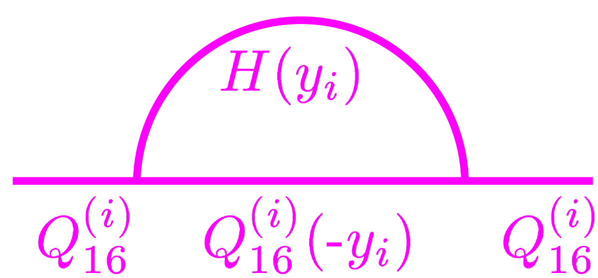
$$\Longrightarrow \text{Higgsino mass} \begin{pmatrix} 0 & \langle H(-z)\rangle_A \\ \langle H(-z)\rangle_A & m_{10} \end{pmatrix}$$

Gaugino mass



Physical sector mass





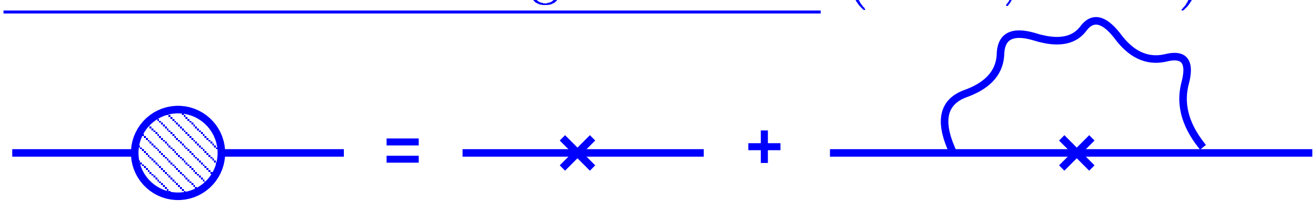
only ν_R gets mass

Gauge symmetry breaking

$$\{SO(10)\}^3 \to SO(10)$$
$$\langle Q_{16}^{(i)}(y_i) Q_{\overline{16}}^{(i)}(\text{-}y_i)\rangle = m_{ij}$$

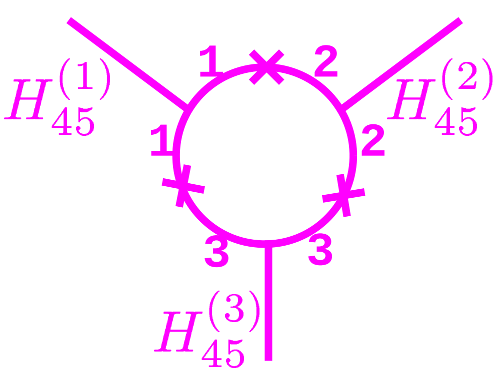
D-term can break the supersymmetry and it will break the powerful non-renormalization theorem (N. Seiberg) also. This is the origin of “Flavor physics”. How do we do it ? Non-perturvative effect must be considered.

Self-consistent mass generations (NJL, BCS)



$$m_{ij} \quad = \quad m_i \delta_{ij} \quad + \quad \Sigma(m_{ij})$$
$$\Gamma(\varphi) \quad = \quad S(\varphi) + \frac{1}{2} \mathrm{tr} \log \left[\frac{\delta^2 S}{\delta\varphi_i \delta\varphi_j} \Big/ \frac{\delta^2 S}{\delta\varphi_i \delta\varphi_j}\Big|_{\varphi=0} \right]$$

$SO(10) \to SU(3) \times SU(2) \times U(1)$



$$\mathrm{tr}\left(H_{45}^{(1)} H_{45}^{(2)} H_{45}^{(3)}\right) \neq 0$$

etc.

In our case, this takes the form

$$\Gamma_{\text{mass}}(\varphi) = S(\varphi) - \frac{1}{2}\text{Tr}\int d\Omega d\Omega' \times$$
$$\Big[K^{-1}(\Omega-\Omega')_{\varphi\varphi}a_{\varphi V}(\Omega')K^{-1}(\Omega'-\Omega)_{VV}a_{V\varphi}(\Omega)$$
$$+ \quad K^{-1}(\Omega-\Omega')_{\varphi V}a_{V\varphi}(\Omega')K^{-1}(\Omega'-\Omega)_{\varphi V}a_{V\varphi}(\Omega)\Big]$$

where

$$d\Omega \equiv dx d^2\theta d^2\bar{\theta}, \quad \frac{\delta^2 S}{\delta\varphi_i \delta\varphi_j} \equiv \overbrace{K_{ij}}^{L_0} + \overbrace{a_{ij}}^{L_{\text{int}}} .$$

For example,

$$K_{V\varphi} =$$
$$\begin{pmatrix} -\Box P_T + \frac{1}{2}m_V^2 - \xi(P_1+P_2)\Box & -2gYA^\dagger(y) & 0 \\ 0 & -\frac{m}{4\Box}DD & 1+gYD\theta^2\bar{\theta}^2 \\ -2gYA(-y) & 1-gYD\theta^2\bar{\theta}^2 & -\frac{m}{4\Box}\bar{D}\bar{D} \end{pmatrix}$$

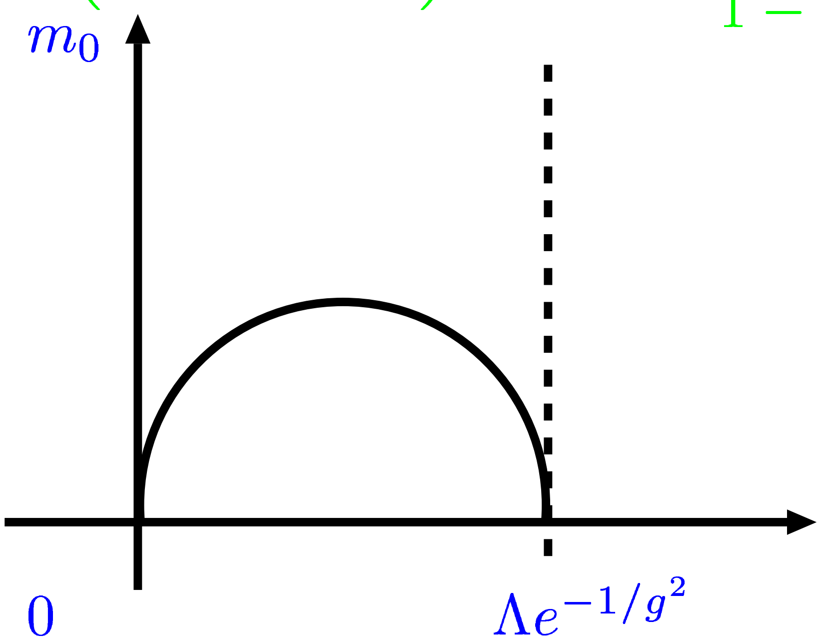
etc.

♠ Prototype computation

$H_{10}^{(1)}$ case

$$m = m_0 + g^2 m \log\frac{\Lambda}{m}$$

$$m_0 = m\left(1 - g^2\log\frac{\Lambda}{m}\right) \quad \text{or} \quad m = \frac{m_0}{1 - g^2\log\frac{\Lambda}{m}}$$



$m_0 = 0 \quad m = 0$ or, $m = \Lambda e^{-1/g^2}$

$\Longrightarrow$ Hierarchical

may be relevant for doublet-triplet splitting

$Q_{16}^{(i)}(y_i)$ case

$$m_{ij} = m_0\delta_{ij} + g_i \left(m \log \frac{\Lambda}{m} \right)_{ij} g_j \quad (i = 1,\ ,2,\ 3)$$

approximations

$$m_{ij} = m_0\delta_{ij} + \overline{m}_{ij}\,, \qquad |\overline{m}_{ij}| \leq |m_0|$$

$$x_{ij} \equiv \frac{\overline{m}_{ij}}{m_0}\,, \qquad \log \frac{\Lambda}{m_0} \equiv \lambda$$

Then

$$x_{ij} = \lambda g_i g_j \delta_{ij} + (\lambda - 1) g_i g_j x_{ij} - \frac{1}{2} g_i g_j \left(x^2 \right)_{ij}$$

assume $|g_1| > |g_2| > |g_3|\,, \quad \lambda \gg 1$

then $\quad x_{11} \simeq 1/g_1$

For

$$g_2 \geq \frac{1}{(\lambda-1)g_1 + 1/2}\,, \quad g_3 \leq \frac{1}{(\lambda-1)g_1 + 1/2}$$

$$x_{22} \simeq \lambda g_2^2\,, \quad x_{33} \simeq \lambda g_3^2$$

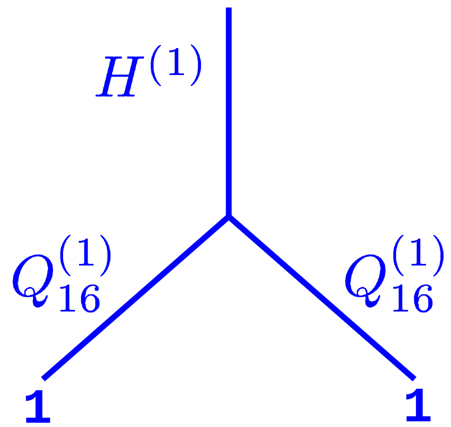
$$x_{12}^2 = \frac{4}{g_1 g_2 g_3^2} \left\{ 1 - (\lambda - 1) g_1 g_3 + \frac{1}{2} g_1 g_3 (x_{11} + x_{33}) \right\}$$

$$x_{13}^2 = \frac{4}{g_1 g_3 g_2^2} \left\{ 1 - (\lambda - 1) g_1 g_2 + \frac{1}{2} g_1 g_2 (x_{11} + x_{22}) \right\}$$

$$x_{23}^2 = x_{12}^2 x_{13}^2 \cdot \frac{1}{2} g_2^2 g_3^2$$

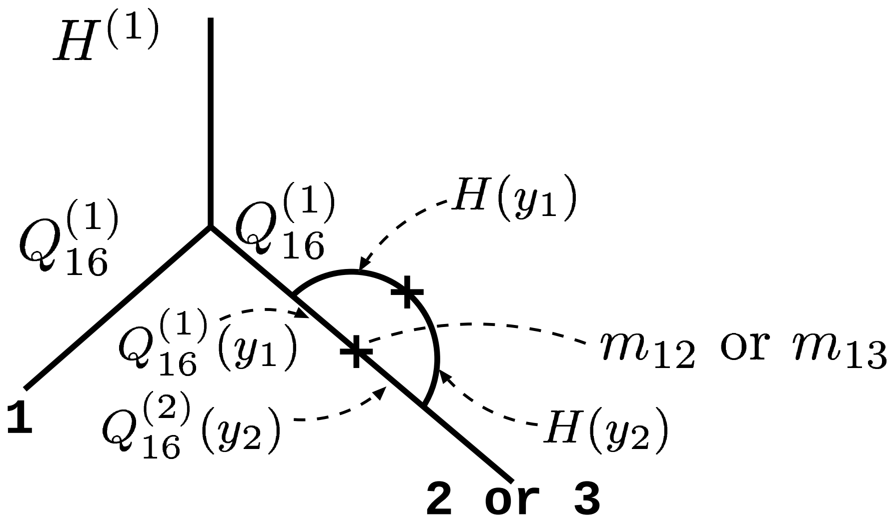
x_{12} real, $\quad x_{13},\ x_{23}$ imaginary $\implies$ CP violation

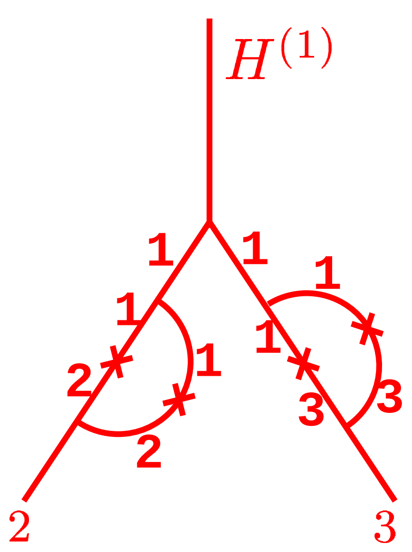
6 Quark-lepton mass matrix and mixing matrix calculation

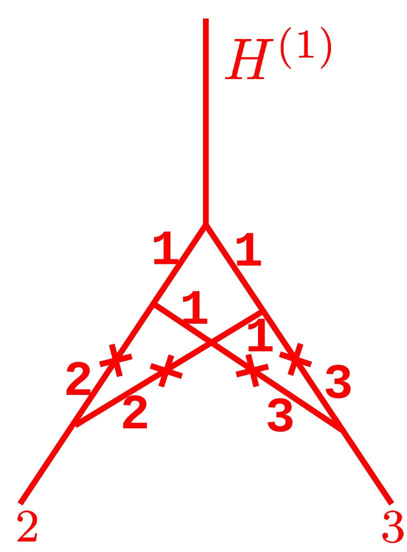


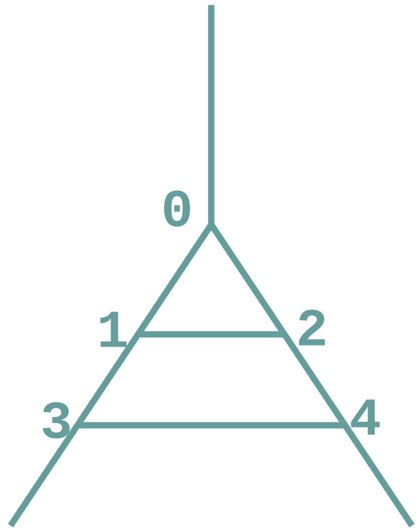
$gQ_{16}^{(1)}Q_{16}^{(1)}H^{(1)}$ is the only coupling

Others can be calculated in the following way:









does not exist

(Proof)

0	H_{10}	Q_{16}	Q_{16}
1	H_{-y}	Q_{16}	$Q_{16}(y)$
2	H_{-y}	$Q_{\overline{16}}$	$Q_{16}(y)$
3	H_{y}	Q_{16}	$Q_{(\overline{16},-y)}$
4	H_{y}	Q_{16}	$Q_{(\overline{16},-y)}$

Mass matrices for U, D, L, ν_D have all

$$\begin{pmatrix} 1+e & c & d \\ a & \alpha & \gamma \\ b & \beta & \delta \end{pmatrix} \quad \text{form}$$

$$\text{with} \begin{cases} e, a, b, c, d & \cdots \ \text{first order} \\ \alpha, \beta, \gamma, \delta & \cdots \ \text{second order} \end{cases}$$

ν_M has

$$\simeq \begin{pmatrix} \Delta_{11} & & \\ & \Delta_{22} & \\ & & \Delta_{33} \end{pmatrix} \quad \text{form}$$

$\Longrightarrow$ see-saw is built in

KM matrix is given by

$$\begin{pmatrix} 1, & -|(\chi^+_-, V_{+d})|e^{i\varphi} - \bar{\kappa}^d_{+-} + \bar{\kappa}^u_{+-}, & -\bar{\kappa}^d_{+0} + \bar{\kappa}^u_{+0} + \bar{\kappa}^d_{-0}|(\chi^+_-, V_{+d})|e^{i\varphi} \\ |(\chi^+_-, V_{+d})|e^{-i\varphi} + \kappa^d_{+-} - \kappa^u_{+-}, & 1 - \frac{1}{2}|\kappa^u_{-0} - \kappa^d_{-0}|^2, & -\bar{\kappa}^d_{-0} + \bar{\kappa}^u_{-0} \\ \kappa^d_{+0} - \kappa^u_{+0} - \kappa^u_{-0}|(\chi^+_-, V_{+d})|e^{-i\varphi}, & \kappa^d_{-0} - \kappa^u_{-0}, & 1 - \frac{1}{2}|\kappa^u_{-0} - \kappa^d_{-0}|^2 \end{pmatrix}$$

where

$$V_+ = \begin{pmatrix} 1 \\ a/e \\ b/e \end{pmatrix}, \quad V_- = \frac{1}{\sqrt{|a|^2 + |b|^2}} \begin{pmatrix} (|a|^2 + |b|^2)/|e| \\ -e^{i\varphi} a \\ -e^{i\varphi} b \end{pmatrix}$$

$$V_0 = \frac{1}{\sqrt{|a|^2 + |b|^2}} \begin{pmatrix} 0 \\ -\bar{b} \\ -\bar{a} \end{pmatrix}, \quad \bar{e} = |e| e^{i\varphi}$$

$$\chi_\pm \equiv V_{\pm u} - V_{\pm d}$$

$$\kappa_{ik} \equiv \frac{1}{\lambda_i - \lambda_k} \langle V_k | H_1 | V_i \rangle, \quad H_1 = \begin{pmatrix} e & c & d \\ a & 0 & 0 \\ b & 0 & 0 \end{pmatrix}$$

$$\lambda_+ = |1 + e|^2 + |a|^2 + |b|^2 + |c|^2 + |d|^2$$

$$\lambda_- = \frac{(|a|^2 + |b|^2)(|c|^2 + |d|^2)}{|1 + e|^2 + |a|^2 + |b|^2 + |c|^2 + |d|^2}$$

$$\lambda_0 = \frac{1}{|c|^2 + |d|^2} \left\{ |a\alpha - c\gamma|^2 + |d\beta - c\delta|^2 \right\} + \text{“second order”}$$

Neutrino mixings

$$M = m_D \begin{pmatrix} \Delta_{11}^{-1} & & \\ & \Delta_{22}^{-1} & \\ & & \Delta_{33}^{-1} \end{pmatrix} m_D^t$$

$$\text{matrix} = \begin{pmatrix} \cos\theta \frac{\lambda_+^*}{|\lambda_+|}, & \cos\theta \frac{(a^* y - b^* x)}{1+e^*}, & \cos\theta \frac{(a^* x^* + b^* y^*)}{1+e^*} \\ \sin\theta \frac{\lambda_+^*}{|\lambda_+|}, & \cos\theta \cot\theta \frac{(-a^* y + b^* x)}{1+e^*}, & \cos\theta \cot\theta \frac{(-a^* x^* - b^* y^*)}{1+e^*} \\ \frac{1}{N_+} \cot\theta \frac{(-by - ax)}{|1+e|}, & \cot\theta \frac{(-by-ax)}{|1+e|}, & \cot\theta \frac{(-bx^* - ay^*)}{|1+e|} \end{pmatrix}$$

Here $\quad \cos\theta = \frac{1}{\sqrt{1 + (|a|^2 + |b|^2)/|1+e|^2}}$

$$\lambda_+ \simeq 1 + c^{*2} + d^{*2}, \quad x \simeq -\frac{1}{N_0} \{b + c\beta + d\delta\}$$

$$\lambda_- \simeq -\frac{N_0^2}{1 + c^2 + d^2}, \quad y \simeq \frac{1}{N_0} \{a + c\alpha + d\gamma\}$$

$$N_+^2 = \frac{|\lambda_+|^2}{N_0^2} + |x|^2 + |y|^2, \quad x^2 + y^2 = 1$$

$$\begin{aligned} c^2 \Delta_{11} \Delta_{22}^{-1} &\longrightarrow c^2 \quad , \quad & d^2 \Delta_{11} \Delta_{33}^{-1} &\longrightarrow d^2 \\ \alpha^2 \Delta_{22}^{-1} &\longrightarrow \alpha^2 \quad , \quad & \beta^2 \Delta_{22}^{-1} &\longrightarrow \beta^2 \\ \gamma^2 \Delta_3^{-1} &\longrightarrow \gamma^2 \quad , \quad & \delta^2 \Delta_{33}^{-1} &\longrightarrow \delta^2 \end{aligned}$$

7 Conclusion

(1) M-theory

$\longleftrightarrow$ renormalizable membrane theory ?

(2) Vacuum problem

It is important to make sure it contains a physically acceptable one

$\Longrightarrow$ phenomenology

(3) A candidate vacuum

$\longrightarrow$ lens space + discrete symmetry

(4) $\{SO(10)\}^3 \times (U(1) \times U(1))$ model

(5) Various hierarchy mechanisms

1. membrane instanton
2. see-saw
3. $\Delta \sim e^{-1/g^2}$

(6) CKM, neutrino mixing are calculable

(7) New physics