LEPTONS AND QUARKS BETWEEN BRANES AND BULK*

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OUTLINE

(1) Gauge Unification in 6D

(2) Flavour Mixing and Seesaw Mechanism

(3) Proton Decay

(4) Towards E_8 in Higher Dimensions

(1) Gauge Unification in 6D

Grand unified theories (GUTs) are natural extension of the standard model; quarks and leptons form SU(5) multiplets (Georgi, Glashow '74),

$$\mathbf{10} = (q_L, u_R^c, e_R^c) \ , \quad \mathbf{5}^* = (d_R^c, l_L) \ , \quad (\mathbf{1} = \nu_R) \ ,$$

or SU(4)×SU(2)×SU(2) multiplets (Pati, Salam '74),

$$(\mathbf{4}, \mathbf{2}, \mathbf{1}) = (q_L, l_L) \ , \quad (\mathbf{4}^*, \mathbf{1}, \mathbf{2}) = (u_R^c, d_R^c, \nu_R^c, e_R^c) \ ;$$

all quarks and leptons of one generation are unified in a single multiplet in the GUT group SO(10) (Georgi; Fritsch, Minkowski '75),

$$\mathbf{16} = \mathbf{10} + \mathbf{5}^* + \mathbf{1} = (\mathbf{4}, \mathbf{2}, \mathbf{1}) + (\mathbf{4}^*, \mathbf{1}, \mathbf{2}) \ .$$

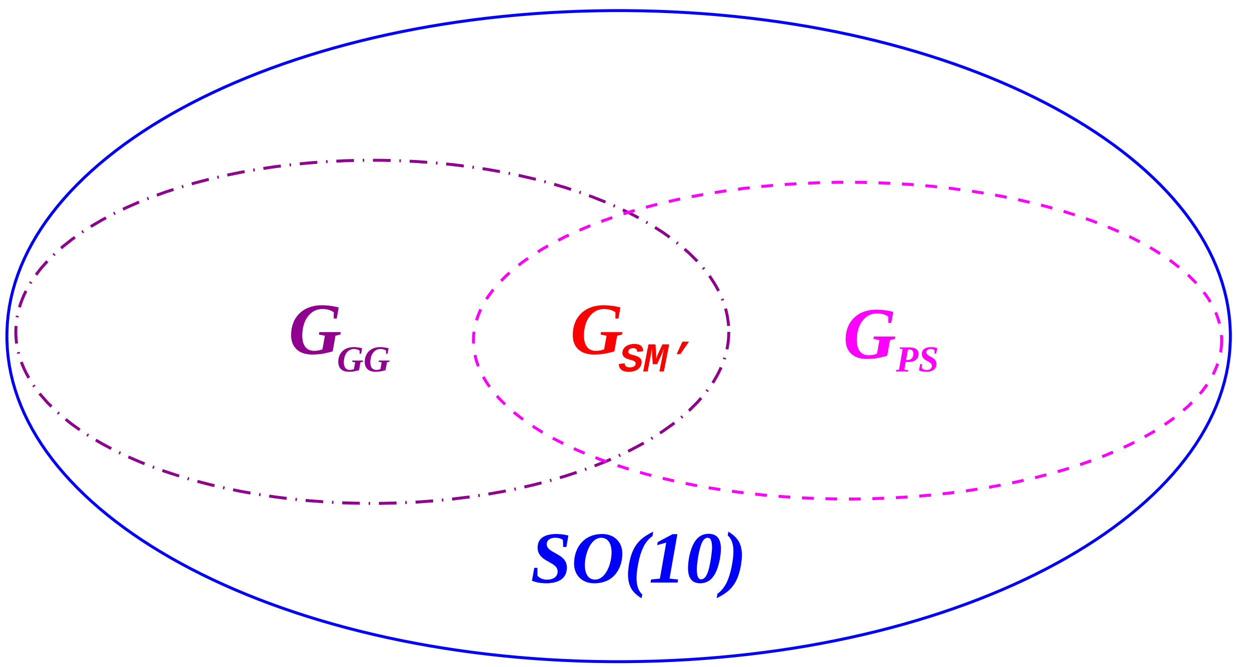
Orbifold compactifications are familiar from string theories (Dixon, Harvey, Vafa, Witten '85); recent application to GUT field theories (Kawamura; Altarelli, Feruglio; Hall, Nomura; Hebecker, March-Russell;...). GUT symmetry breaking automatically yields the required doublet-triplet splitting of Higgs fields. Several successful models SU(5) models have been constructed in 5 dimensions; 6 dimensions are attractive for the breaking of SO(10) (ABC; Hall, Nomura, Okui, Smith;...), but not unavoidable (Dermisek, Mafi; Kim, Raby;...).

Consider SO(10) gauge theory in 6D with N=2 supersymmetry. The gauge fields $V_M(x,y,z)$, with $M = \mu, 5, 6$, $x^5 = y$, $x^6 = z$, and the gauginos λ_1, λ_2 can be grouped into vector and chiral multiplets of the unbroken N=1 supersymmetry in 4D,

$$V = (V_\mu, \lambda_1) \ , \quad \Sigma = (V_{5,6}, \lambda_2) \ .$$

V and Σ are matrices in the adjoint representation of SO(10). Symmetry breaking is achieved by compactification on $T^2/(Z_2^I \times Z_2^{PS} \times Z_2^{GG})$. The discrete symmetries Z_2 break the extended supersymmetry; they also break the SO(10) bulk gauge group to the different subgroups

$$G_{PS} = SU(4) \times SU(2) \times SU(2) \, , \quad G_{GG} = SU(5) \times U(1)_X \, .$$



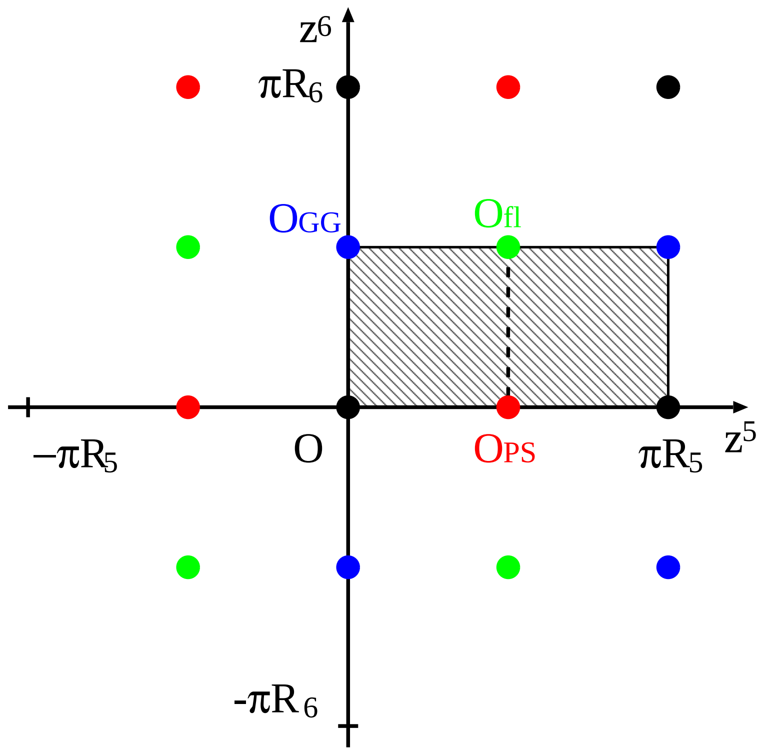
The breaking is localized at different points in the extra dimensions, $O = (0,0)$, $O_{PS} = (\pi R_5/2, 0)$ and $O_{GG} = (0, \pi R_6/2)$,

$$\begin{aligned} P_I V(x,-y,-z) P_I^{-1} &= \eta_I V(x,y,z) \;, \\ P_{PS} V(x,-y+\pi R_5/2,-z) P_{PS}^{-1} &= \eta_{PS} V(x,y+\pi R_5/2,z) \;, \\ P_{GG} V(x,-y,-z+\pi R_6/2) P_{GG}^{-1} &= \eta_{GG} V(x,y,z+\pi R_6/2) \;. \end{aligned}$$

Here $P_I = I$, the matrices P_{PS} and P_{GG} break SO(10) to G$_{PS}$ and G$_{GG}$, and the parities are chosen as $\eta_I = \eta_{PS} = \eta_{GG} = +1$. The extended supersymmetry is broken by choosing in the corresponding equations for Σ all parities $\eta_i = -1$. There is a fourth fixpoint at $O_{fl} = (\pi R_5/2, \pi R_6/2)$, which is obtained by combining the three discrete symmetries Z_2, Z_2^{PS} and Z_2^{GG} defined above,

$$P_{fl} V(x,-y+\pi R_5/2,-z+\pi R_6/2) P_{fl}^{-1} = +V(x,y,+\pi R_5/2, z+\pi R_6/2) \;.$$

The fourth unbroken subgroup at the fixpoint O_{fl} is flipped SU(5), i.e. G_{fl}=SU(5)$'\times$U(1)$'$. The physical region is a 'pillow' with the four fixpoints as corners.



The unbroken gauge group of the effective 4D theory is given by the intersection of the SO(10) subgroups at the fixpoints. In this way one obtains the standard model group with an additional U(1) factor,

$$G_{SM'} = SU(3) \times SU(2) \times U(1)_Y \times U(1)_X \ .$$

The difference of baryon and lepton number is the linear combination $B - L = \sqrt{\frac{16}{15}} Y - \sqrt{\frac{8}{5}} X$. The zero modes of the vector multiplet V form the gauge fields of $G_{SM'}$.

The vector multiplet V is a **45**-plet of SO(10) which has an irreducible anomaly in 6 dimensions. It is related to the irreducible anomalies of hypermultiplets in the fundamental and the spinor representations,

$$a(\mathbf{45}) = -2a(\mathbf{10}) \ , \quad a(\mathbf{16}) = a(\mathbf{16}^*) = -a(\mathbf{10}) \ .$$

Anomaly cancellation requires two **10** hypermultiplets, H_1 and H_2. Possible choice of SU(2) doublets as zero modes, i.e., doublet-triplet splitting,

$$H_1^c = H_d \ , \quad H_2 = H_u \ ,$$

yields wanted Higgs doublets of MSSM. Flat direction, $\langle H_1^c \rangle = \langle H_2 \rangle = v$, may be stabilized at electroweak scale by supersymmetry breaking at brane.

$B - L$ breaking can be achieved by flat direction of additional **16** hypermultiplets Φ^c, Φ with zero modes N^c, N,

$$\langle N^c \rangle = \langle N \rangle = v_N \ ,$$

with $v_N \gg v$ fixed by brane superpotential. Anomaly cancellation requires two **10** hypermultiplets H_3, H_4. Colour triplet zero modes (D, G^c) and (D^c, G) aquire masses $\mathcal{O}(v_N)$ from brane superpotential.

(2) Flavour mixing and seesaw mechanism

How can matter be introduced ? Guiding principles: anomaly cancellation, embedding in E_8. Quarks and leptons cannot be bulk fields, too many **16**-plets required, they have to be brane fields.

As an example, place ψ_1 at O_{GG}, ψ_2 at O_{fl} and ψ_3 at O_{PS}. The three 'families' are separated by distances large compared to the 6D cutoff scale M_*; they can only have diagonal Yukawa couplings with the bulk Higgs fields, direct mixings are exponentially suppressed.

However, brane fields can mix with bulk zero modes without suppression. E_8 embedding allows two additional **10** hypermultiplets H_5, H_6, together with two **16**'s, ϕ and ϕ^c, with zero modes,

$$L = \begin{pmatrix} \nu_4 \\ e_4 \end{pmatrix} , \quad L^c = \begin{pmatrix} \nu_4^c \\ e_4^c \end{pmatrix} , \quad G_5^c = d_4^c , \quad G_6 = d_4 .$$

Mixings take place only among left-handed leptons and right-handed down-quarks (cf. lopsided SU(5) models). This leads to a characteristic pattern of mass matrices.

Masses and mixings are determined by brane superpotentials. Allowed terms are restricted by R-invariance and an additional $U(1)_{\tilde{X}}$ symmetry. H_1, H_2, Φ and Φ^c, which aquire a vacuum expectation value, have R-charge zero. All matter fields have R-charge one. The **16**-plets ψ_i and ϕ form a quartet $(\psi_\alpha) = (\psi_i, \phi)$, $\alpha = 1 \ldots 4$. Most general brane superpotential, for normalized bulk fields, up to quartic interactions (23 terms),

$$\begin{aligned} W &= M^l_\alpha \psi_\alpha \phi^c + \frac{1}{2} h^{(1)}_{\alpha\beta} \psi_\alpha \psi_\beta H_1 + \frac{1}{2} h^{(2)}_{\alpha\beta} \psi_\alpha \psi_\beta H_2 \\ &\quad + \frac{1}{2} \frac{h^N_{\alpha\beta}}{M_*} \psi_\alpha \psi_\beta \Phi^c \Phi^c + \ldots \ ; \end{aligned}$$

$M_* > 1/R_{5,6} \sim \Lambda_{GUT}$ is the cutoff of the 6d theory. On the different branes the Yukawa couplings $h^{(1)}$ and $h^{(2)}$ split into h^d, h^e and h^u, h^D, respectively.

$B - L$ breaking yields masses $\mathcal{O}(v_N)$ for colour triplet bulk zero modes. After electroweak symmetry breaking, $\langle H_1^c \rangle = v_1$, $\langle H_2 \rangle = v_2$, all zero modes aquire mass terms,

$$\begin{aligned} W &= d_\alpha m^d_{\alpha\beta} d^c_\beta + e^c_\alpha m^e_{\alpha\beta} e_\beta + n^c_\alpha m^D_{\alpha\beta} \nu_\beta \\ &\quad + u^c_i m^u_{ij} u_j + \frac{1}{2} n^c_i M_{ij} n^c_j \ . \end{aligned}$$

Some of the mass matrix elements are equal due to GUT relations on the corresponding brane, others are not; e.g. $m^e_{11} = m^d_{11}$, $m^e_{22} \neq m^d_{22}$, $m^D_{22} = m^u_{22}$, etc.

m^d, m^e and m^D are 4×4 matrices, e.g.,

$$m^e = \begin{pmatrix} h^d_{11} v_1 & 0 & 0 & h^e_{14} v_1 \\ 0 & h^e_{22} v_1 & 0 & h^e_{24} v_1 \\ 0 & 0 & h^d_{33} v_1 & h^e_{34} v_1 \\ M^l_1 & M^l_2 & M^l_3 & M^l_4 \end{pmatrix} ;$$

m^u and m^N are diagonal 3×3 matrices,

$$m^u = \begin{pmatrix} h^u_{11} v_2 & 0 & 0 \\ 0 & h^u_{22} v_2 & 0 \\ 0 & 0 & h^u_{33} v_2 \end{pmatrix} , \quad m^N = \begin{pmatrix} h^N_{11} \frac{v_N^2}{M_*} & 0 & 0 \\ 0 & h^N_{22} \frac{v_N^2}{M_*} & 0 \\ 0 & 0 & h^N_{33} \frac{v_N^2}{M_*} \end{pmatrix} .$$

The diagonal elements satisfy four GUT relations which correspond to the unbroken SU(5), flipped SU(5) and Pati-Salam subgroups of SO(10).

General pattern of quark and lepton mass matrices, assuming universal Yukawa couplings at each fixpoint,

$$\frac{1}{\tan\beta}\, m^u \sim \frac{v_1 M_*}{v_N^2}\, m^N \sim \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix} ,$$

$$m^d \sim m^e \sim m^D \sim \begin{pmatrix} \mu_1 & 0 & 0 & \widetilde{\mu}_1 \\ 0 & \mu_2 & 0 & \widetilde{\mu}_2 \\ 0 & 0 & \mu_3 & \widetilde{\mu}_3 \\ \widetilde{M}_1 & \widetilde{M}_2 & \widetilde{M}_3 & \widetilde{M}_4 \end{pmatrix} ,$$

with $\mu_i, \widetilde{\mu}_i = \mathcal{O}(v_1)$ and $\widetilde{M}_i = \mathcal{O}(\Lambda_{GUT})$. Phenomenology requires $\mu_i, \widetilde{\mu}_i$ to be hierarchical, the GUT mixings $\widetilde{M}_i$ are assumed to be non-hierarchical.

Up-quark and heavy neutrino masses,

$$\mu_1 : \mu_2 : \mu_3 \sim m_u : m_c : m_t \sim M_1 : M_2 : M_3 \ .$$

Down-quark masses and CKM mixings for large $\tan\beta = v_2/v_1 \simeq 50$, such that $h^d_{33} \simeq h^u_{33}$; dominated by off-diagonal elements $\widetilde{\mu}_i$, since down-quark hierarchy much smaller than upp-quark hierarchy,

$$\mu_1 \ll \widetilde{\mu}_1 \ , \quad \mu_2 \ll \widetilde{\mu}_2 \ , \quad \mu_3 \sim \widetilde{\mu}_3 \ .$$

Determination of parameters,

$$m_b \simeq \widetilde{\mu}_3 \ , \quad m_s \simeq \widetilde{\mu}_2 \ , \quad V_{us} = \Theta_c \sim \frac{\widetilde{\mu}_1}{\widetilde{\mu}_2} \ ;$$

Predictions of mixing angles from diagonalization of m^d,

$$V_{cb} \sim \frac{m_s}{m_b} \simeq 2 \times 10^{-2} , \quad V_{ub} \sim \Theta_c \frac{m_s}{m_b} \simeq 4 \times 10^{-3} ,$$

and down-quark mass,

$$\frac{m_d}{m_s} \sim \gamma \, \Theta_c \simeq \; 0.03 , \quad \gamma \sim \frac{\mu_2}{\widetilde{\mu}_2} \sim \frac{m_c m_b}{m_t m_s} \sim 0.1 ,$$

consistent with data within factor of two. Charged lepton masses also o.k.; small electron mass, $m_e/m_\mu \simeq 0.1 \; m_d/m_s$, not a problem since no SU(5) mass relation on flipped-SU(5) brane.

The light neutrino masses are given by the seesaw relation

$$m_\nu = -m^{DT} \frac{1}{M^N} m^D .$$

The structure of the charged lepton and the Dirac neutrino mass matrices is the same; both matrices lead to large mixings between the 'left-handed' states. However, due to seesaw mechanism, mismatch and large remaining leptonic MNS mixings. m_ν in basis where m^e is hierarchical,

$$m_\nu \sim \begin{pmatrix} \gamma^2 & \gamma & \gamma \\ \gamma & 1 & 1 \\ \gamma & 1 & 1 \end{pmatrix} m_3 \ ;$$

result familiar from lopsided SU(5) models (Sato,Yanagida; Irges, Lavignac, Ramond; Altarelli, Feruglio;...); characteristic prediction is a rather large 1-3 mixing angle, $\Theta_{13} \sim \gamma \sim 0.1$. Coefficients $\mathcal{O}(1)$ are consistent with 'sequential heavy neutrino dominance' (N_3), yielding large 2-3 mixing, $\sin 2\Theta_{23} \sim 1$.

With $m_3 \simeq \sqrt{\Delta m^2_{atm}} \sim m_t^2/M_3$ the heavy Majorana masses are $M_3 \sim 10^{15}$ GeV, $M_2 \sim 3 \times 10^{12}$ GeV and $M_1 \sim 10^{10}$ GeV; the second neutrino mass is $m_2 \sim 0.01$ eV. One also abtains the CP-asymmetry $\varepsilon_1 \sim 0.1\ M_1/M_3 \sim 10^{-6}$ and the effective neutrino mass $\widetilde{m}_1 = (m^{D\dagger} m^D)_{11}/M_1 \sim 0.2\, m_3 \sim 0.01$ eV, i.e. the standard parameters of thermal leptogenesis (WB, Plümacher). Neutrino phenomenology is fixed in terms of quark masses and mixings.

Fundamental question in flavour physics: different mass and mixing patterns for quarks and neutrinos, compatibility with grand unification; answer in 6D orbifold GUT:

- MNS mixings are large because neutrinos are mixtures of brane and bulk states, which is unrelated to the hierarchy of quark and charged lepton masses.

- CKM mixings are small because left-handed down-quarks are brane states. The large mixings of right-handed down quarks, together with the down-quark mass hierarchy, implies small mixings for left-handed quarks.

- Neutrinos have a small mass hierarchy because of the seesaw mechanism and the mass relations $m^d \sim m^D$, $m^u \sim m^N$; the 'squared' down-quark hierarchy is almost canceled by the larger up-quark hierarchy,

$$\frac{m_1}{m_3} \sim \left(\frac{m_d}{m_b}\right)^2 \frac{m_t}{m_u} \sim 0.1 \ .$$

Basic mechanism: mixing with split multiplets breaks GUT relations for mass matrices; mass hierarchies have 'geometric origin' and are not explained by abelian or non-abelian flavour symmetry.

(3) Proton decay

As in 5D SU(5) models, dangerous dimension-5 operators for proton decay are absent; the dimension-6 operator has an interesting flavour structure due to the localization of brane fields (Nomura; Hebecker, March-Russell); in the 6D SO(10) model only the fields localized on the SU(5) brane contribute,

$$\mathcal{L}_{eff} \quad = \quad \frac{g_4^2}{M_{\mathcal{X}}^2} \epsilon_{\alpha\beta\gamma} \left(e_1^c u_{1\alpha}^c \overline{q}_{1\beta} \overline{q}_{1\gamma} - d_{1\alpha}^c u_{1\beta}^c \overline{q}_{1\gamma} \overline{l}_1 \right) \; .$$

The summation over Kaluza-Klein states is logarithmically divergent and depends on the cutoff $M_* \sim 10^{17}$ GeV ($R_5 = R_6 = 1/M_c$),

$$\sum_{n,m=0}^{\infty} \frac{1}{M_{\mathcal{X}}^2(n,m)} \simeq \frac{1}{M_c^2} \left(\frac{\pi}{8} \ln \left(\frac{M_*}{M_c} \right) + C \right) \; .$$

Branching ratios depend on overlap of SU(5)-brane states with mass eigenstates, $d_1^c = U^d_{R1i}\hat{d}_i^c$, $e_1 = U^e_{L1i}\hat{e}_i$, etc, explicitly known in 6D SO(10) model. Bound on compactification scale from dominant decay mode,

$$\frac{1}{\Gamma(p \to e^+\pi^0)} = (8 \times 10^{34}\ \mathrm{yr}) \left(\frac{M_c}{10^{16}\ \mathrm{GeV}}\right)^4 \left(\ln \frac{M_*}{M_c}\right)^{-2} ;$$

the Super-Kamiokande bound of 5.3×10^{33} yr yields $M_c \geq 0.8 \times 10^{16}$ GeV, close to usual unification scale 2×10^{16} GeV ($M_* \simeq 10^{17}$ GeV).

Branching ratios for 4D SU(5) theory and 6D SO(10) model are different!

	$\pi^0 e^+$	$\pi^0 \mu^+$	$\pi^0 \overline{\nu}$	$K^0 e^+$	$K^0 \mu^+$	$K^+ \overline{\nu}$
BR[%] 6D SO(10)	88	4	7	1	0.1	0.1
BR[%] 4D SU(5)	64	0.6	25	0.2	9	0.8

(4) Towards E_8 in higher dimensions

Our work has partly been motivated by previous attempts to relate quarks and leptons to a coset space G/H, where G is an appropriate simple group and H contains the standard model gauge group (WB, Love, Peccei, Yanagida '82; Ong '83;...; Groot Nibbelink, van Holten). Attractive coset space is $E_8/SO(10)\times SU(3)\times U(1)$ with complex structure

$$\Omega = (\mathbf{16},\mathbf{3})_1 + (\mathbf{16}^*,\mathbf{1})_3 + (\mathbf{10},\mathbf{3}^*)_2 + (\mathbf{1},\mathbf{3})_4 \ .$$

Interpretation of 4D supersymmetric σ-model: three **16** quark-lepton generations, one mirror $\mathbf{16}^*$ generation, additinal **10** Higgs fields and SO(10) singlets; intriguing extension of SM, but phenomenologically too many fields. Possible role of coset for orbifold GUTs unclear; open questions: bulk fields and/or brane fields, spontaneous breaking of E_8, etc ?

Features of bottom-up approach. Consider SO(10) content of E_8 adjoint representation,

$$\mathbf{248} = \mathbf{45} + 4 \times \mathbf{16} + 4 \times \mathbf{16}^* + 6 \times \mathbf{10} + \ldots .$$

Anomaly cancellation of 6D SO(10) model requires 2 $\mathbf{10}$ hypermultiplets; 2 ($\mathbf{10}$ + $\mathbf{16}^*$) hypermultiplets are used for $B-L$ breaking, 2 ($\mathbf{10}$ + $\mathbf{16}^*$) hypermultiplets are needed for flavour mixing. Bulk anomalies of the remaining 4 $\mathbf{16}$'s are not canceled, are they localized on the branes? This would be almost the considered model. Is one $\mathbf{16}$ heavy or decoupled?

Can the 6D SO(10) model be obtained from E_8 super Yang-Mills theory in 10D? What leads to the breaking of E_8 to SO(10)? Which dynamics implies the symmetry breaking at the orbifold fixpoints? Are there consistency conditions which relate brane fields to bulk fields? Does the embedding in the adjoint representation of E_8 make any sense (cf. string theories)? etc.

SUMMARY

Higher dimensions offer elegant way of GUT symmetry breaking; split multiplets are simple explanation of doublet-triplet splitting.

Together with a geometric origin of fermion mass hierarchies, split multiplets also lead to interesting flavour physics.

This motivates further studies on the embedding of 6D SO(10) GUT in higher dimensional E_8 unified theory.