$$\mathcal{L} = m \bar{\nu}_R \nu_L + \frac{M}{2} \nu_R \nu_R + h.c$$

Pauli - Gürsey Transformation

$$\begin{cases} \nu_L \rightarrow \nu = \nu_L + \nu_L^c \\ \nu_R \rightarrow N = \nu_R + \nu_R^c \end{cases}$$

$$(\nu, N) \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \begin{pmatrix} \nu \\ N \end{pmatrix}$$

$$m_\nu = m^2 / M$$

LARGE LEPTON MIXING IN A STRING BRANE WORLD

T. Yanagida

BI-LARGE MIXINGS FOR NEUTRINOS

ATOMOSPHERIC ν OSCILLATION

$$\delta m^2_{32} \simeq (2-3) \times 10^{-3}\, eV^2$$

$$(\sin 2\theta_{23})^2 \simeq \underline{1}$$

Super K

SOLAR ν OSCILLATION

$$\delta m^2_{21} \simeq 7 \times 10^{-5}\, eV^2$$

$$(\sin 2\theta_{12})^2 \simeq \underline{0.8}$$

Super K
SNO
KamLAND

CHOOZ BOUND

$$(\sin 2\theta_{13})^2 < 0.2$$

CHOOZ

WHY LEPTON MIXING
$\gg$ QUARK MIXING ?

GEOMETRIC ORIGIN OF THE LARGE MIXING IN HIGHER DIMENSIONAL SPACE-TIME

Watari . T. Y.

FINE - TUNING PROBLEM IN SUSY GUT's

$$\langle \Sigma(24) \rangle = \begin{pmatrix} 2V & & & & 0 \\ & 2V & & & \\ & & 2V & & \\ & & & -3V & \\ 0 & & & & -3V \end{pmatrix}$$

$$SU(5)_{GUT} \longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y$$

HIGGS MULTIPLETS :

$$H(5) = \begin{pmatrix} H_C \\ H_f \end{pmatrix} \quad ; \quad \begin{pmatrix} \bar{H}_C \\ \bar{H}_f \end{pmatrix} = \bar{H}(5^*)$$

THEIR MASSES :

$$W = M H \bar{H} + \lambda H \Sigma \bar{H}$$

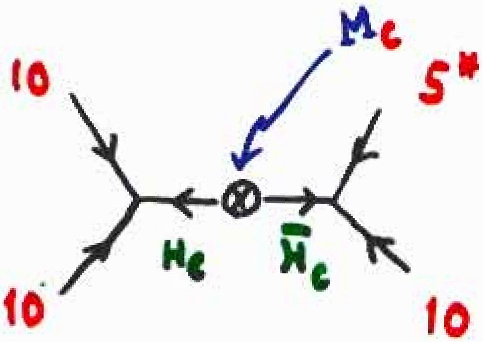
$$
\begin{cases}
M_C = M + 2\lambda V \\
m_f = M - 3\lambda V
\end{cases}
$$

$$V \simeq 10^{16}\ \mathrm{GeV}$$

FINE TUNE : $m_f \lesssim O(100)\ \mathrm{GeV}$

$$\left| \ast 10^{16} - \ast 10^{16} \right| \lesssim 10^{2} \quad ???$$

- D = 5 PROTON DECAY



$$\sim \frac{1}{M_c}$$

Sakai, T.Y.
Weinberg

→ TOO FAST PROTON DECAY ?

Super K

PRODUCT - GROUP UNIFICATION

T.Y. (+ Murayama) (1994)

SUPPOSE THE GUT BREAKING PRODUCES

massless $\nearrow$ $\xi_c(3) + \bar{\xi}_c(3^*)$. THEN, THE COLORED HIGGS

$H_c(3)$ AND $\bar{H}_c(3^*)$ HAVE MASSES TOGETHER

WITH $\bar{\xi}_c$ AND ξ_c.

$$W = M_c H_c \bar{\xi}_c + M_c' \bar{H}_c \xi_c$$

BUT, THE HIGGS DOUBLETS H_f AND $\bar{H}_f$

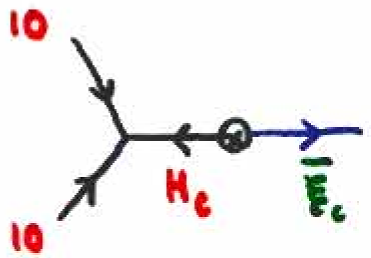
REMAIN MASSLESS, SINCE THEY HAVE NO

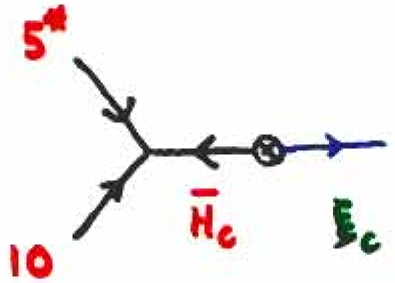
MASS PARTNER.

Masiero et al (1982)

$\{ M H \bar{H}$ IS FORBIDDEN BY A SYMMETRY. $\}$

* NO D=5 PROTON DECAY !!

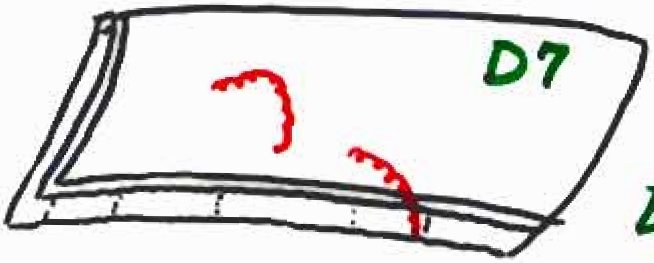




PRODUCT-GROUP UNIFICATION

IN TYPE IIB STRING THEORY

Watari, T.Y. (04)
hep-ph/042160



BRANES

$\mathcal{N}$ D7 BRANES

U(N) GAUGE THEORY

with

16 SUPER-CHARGES REMAIN
IN (7+1) SPACETIME.

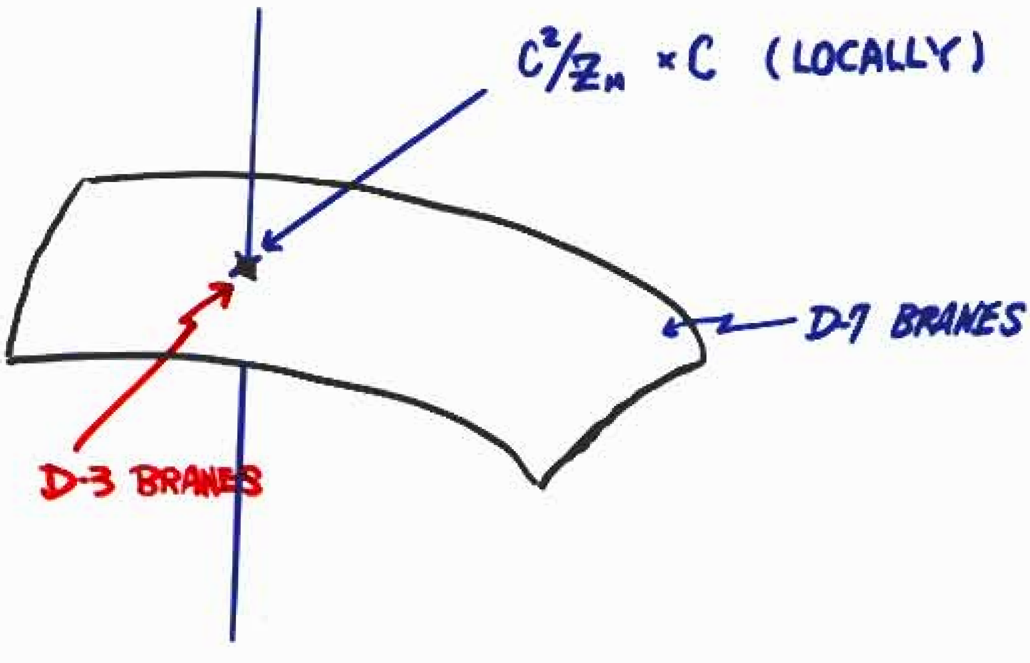
$\left\{ \mathcal{N}=4 \text{ SUSY IN } (3+1) \text{ SPACETIME.} \right\}$

CONSIDER 5 D-7 BRANES:

WE HAVE $U(5)$ GAUGE THEORY.

WE IDENTIFY THE $SU(5)$ SUBGROUP WITH THE $SU(5)$ GUT.

INTRODUCE 3 D-3 BRANES AT A $C^2/\mathbb{Z}_M$ SINGULARITY ON THE D-7 BRANES.



8 SUPER-CHARGES REMAIN ON D-3 BRANES.

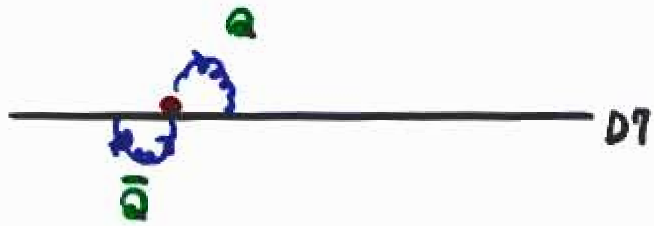
WE HAVE $U(3)$ GAUGE THEORY WITH

$\mathcal{N}=2$ SUSY.

- $U(3)$ GAUGE MULTIPLETS:

D3-D3 OPEN STRINGS

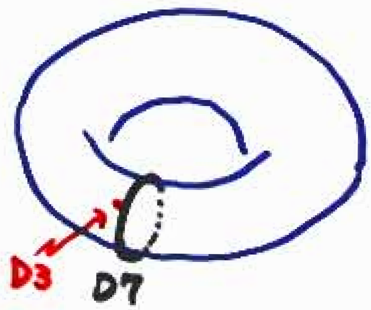
- $Q^{\alpha}_{i} + \bar{Q}_{\alpha}^{i}$ $(3, 5^{*})$ HYPER MULTIPLETS



D3-D7 OPEN STRING

- $SU(5)_{GUT}$ GAUGE MULTIPLETS:

D7-D7 OPEN STRING



Calabi - Yau

$\mathcal{N}=1$ SUSY

$SU(5)_{GUT} \times SU(3) \times U(1)$ THEORY

WITH $\mathcal{N}=1$ SUSY

IN (3+1) DIM. SPACE-TIME.

BUT, LOCALLY WE HAVE $\mathcal{N}=2$ SUSY

$SU(3) \times U(1)$ GAUGE THEORY :

GAUGE MULTIPLETES :

$$\begin{cases} \mathcal{V}^a_{D3} = (V^a, \Phi^a) & a = 1-8 \\ \mathcal{V}^0_{D3} = (V^0, \Phi^0) & \end{cases}$$

HYPER MULTIPLETS

$$Q^a{}_i : (3, 5^*) + \bar{Q}^i{}_a (3^*, 5)$$

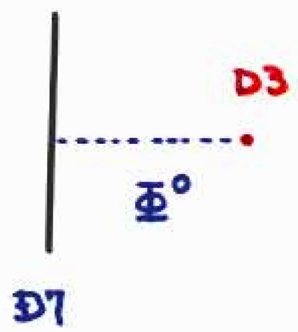
$i = 1-5$

$a = 1-3$

SUPERPOTENTIAL ($\mathcal{N}=2$ SUSY)

$$W = \underbrace{\sqrt{2}\, g_3}_{SU(3)}\ \bar{Q}^i_a\, \Phi^\alpha \cdot (\lambda^\alpha)^a{}_b\, Q^b_i$$

$$+ \sqrt{2}\, \underbrace{g_1}_{U(1)}\ \bar{Q}^i_a\, \Phi^0 (\lambda^0)^a{}_b\, Q^b_i$$



FLAT POTENTIAL FOR Φ^0

WE SUPPOSE THAT SOME DYNAMICS GENERATES ATTRACTIVE FORCE BETWEEN D3 AND D7 TO FORM D3-D7 BOUND STATE.

$\left\{ \begin{array}{l} \text{CONDENSATION OF ANTI-SYMMETRIC } B_{\mu\nu} \\ \text{IS AN EXAMPLE.} \end{array} \right\}$

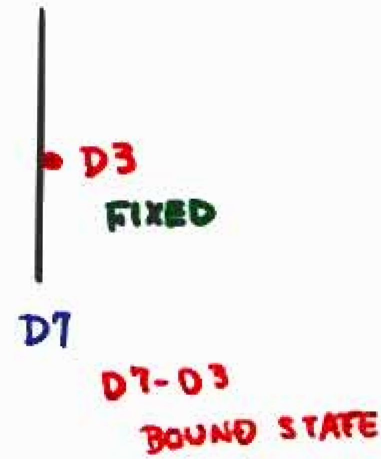
GENERATION OF FI F-TERM ($\mathcal{N}=2$ SUSY)

$$W^{eff}_{FI} = -\sqrt{2}\, g_1 v^2 \Phi^0$$

SUSY-VACUUM

$$\langle \Phi^0 \rangle = 0$$

$$\langle Q^a_{\ i} \rangle = \begin{pmatrix} v & & \\ & v & \\ & & v \\ 0 & & \\ 0 & & \end{pmatrix}$$



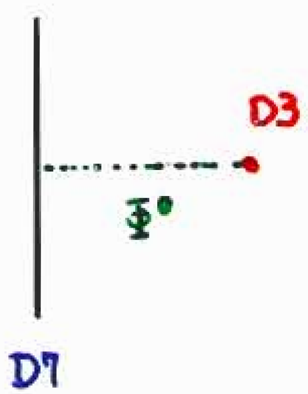
$$\langle \bar{Q}^{\ i}_{a} \rangle = \begin{pmatrix} v & & & 0 & 0 \\ & v & & & \\ & & v & & \end{pmatrix}$$

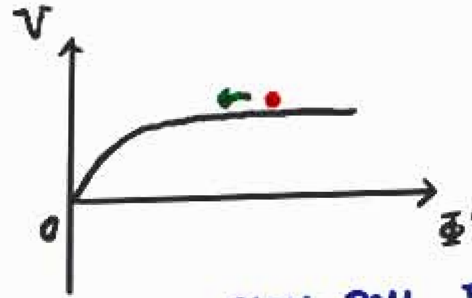
GUT-BREAKING

THERE IS NO FLAT DIRECTION !!

$$SU(5)_{GUT} \times SU(3) \times U(1)$$

$$\longrightarrow SU(3)_c \times SU(2)_L \times U(1) \quad !!!$$

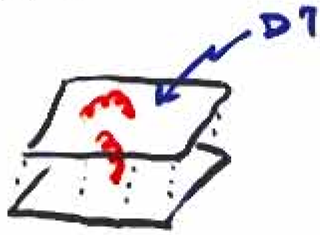




SLOW-ROLL INFLATION OCCURS. Kallosh et al.

Watari, T.Y.

* NOW ADD ONE MORE D-7 BRANE :



WE HAVE $SU(6) \times U(1)$ ON THE D7 BRANES.

WE INTRODUCE NON-TRIVIAL BACKGROUND OF $U(1)$
FIELD STRENGTH.

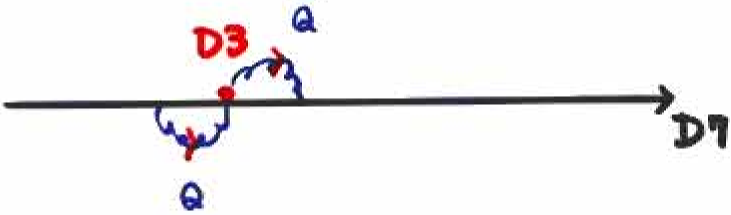
$\int F_{\mu\nu} \neq 0$

$$\lambda = \begin{pmatrix} 1 & & & 0 \\ & \ddots & & \\ & & 1 & \\ 0 & & & -5 \end{pmatrix}$$

$$\rightarrow \quad SU(6) \longrightarrow SU(5)_{GUT} \times U(1)$$

BUT, D3-D7 OPEN STRINGS GIVE MASSLESS
HYPER-MULTIPLETS :

$$\left\{ \begin{array}{ll} Q^a_i : (3, 5^*) & + \quad Q^a_0 \, (3, 1) \\ \bar{Q}^i_a \, (3^*, 5) & + \quad \bar{Q}^0_a \, (3^*, 1) \end{array} \right.$$



AFTER THE GUT BREAKING,

$$SU(5)_{GUT} \times SU(3)_{D3} \times U(1)_{D3} \longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y ,$$

THE $Q^a_0(3, 1)$ AND $\bar{Q}^0_a(3^*, 1)$

REMAIN MASSLESS.

HERE, $\begin{cases} SU(3)_C = SU(3)_{GUT} + SU(3)_{D3} \\ U(1)_Y = U(1)_{GUT} + U(1)_{D3} \\ SU(2)_L = SU(2)_{GUT} \end{cases}$

$$SU(3)_{GUT} \times SU(2)_{GUT} \times U(1)_{GUT} \subset SU(5)_{GUT}$$

$$\begin{cases} Q^a_0 = SU(3)_C \text{-} 3 & \leftarrow \xi^a(3) \\ \bar{Q}^0_a = SU(3)_C \text{-} 3^* & \leftarrow \bar{\xi}_a(3^*) \end{cases}$$

GAUGE COUPLING UNIFICATION :

$$\begin{cases} \dfrac{1}{\alpha_C} = \dfrac{1}{\alpha_{GUT}} + \dfrac{1}{\alpha_{3\,D3}} \\[2ex] \dfrac{1}{\alpha_{2L}} = \dfrac{1}{\alpha_{GUT}} \\[2ex] \dfrac{3/5}{\alpha_Y} = \dfrac{1}{\alpha_{GUT}} + \dfrac{3/5}{\alpha_{1\,D3}} \end{cases}$$

APPROXIMATE UNIFICATION

$$\rightarrow \quad [\alpha_3, \alpha_1]_{D3} \gtrsim (10-100)\ \alpha_{GUT}$$

Fig 1.

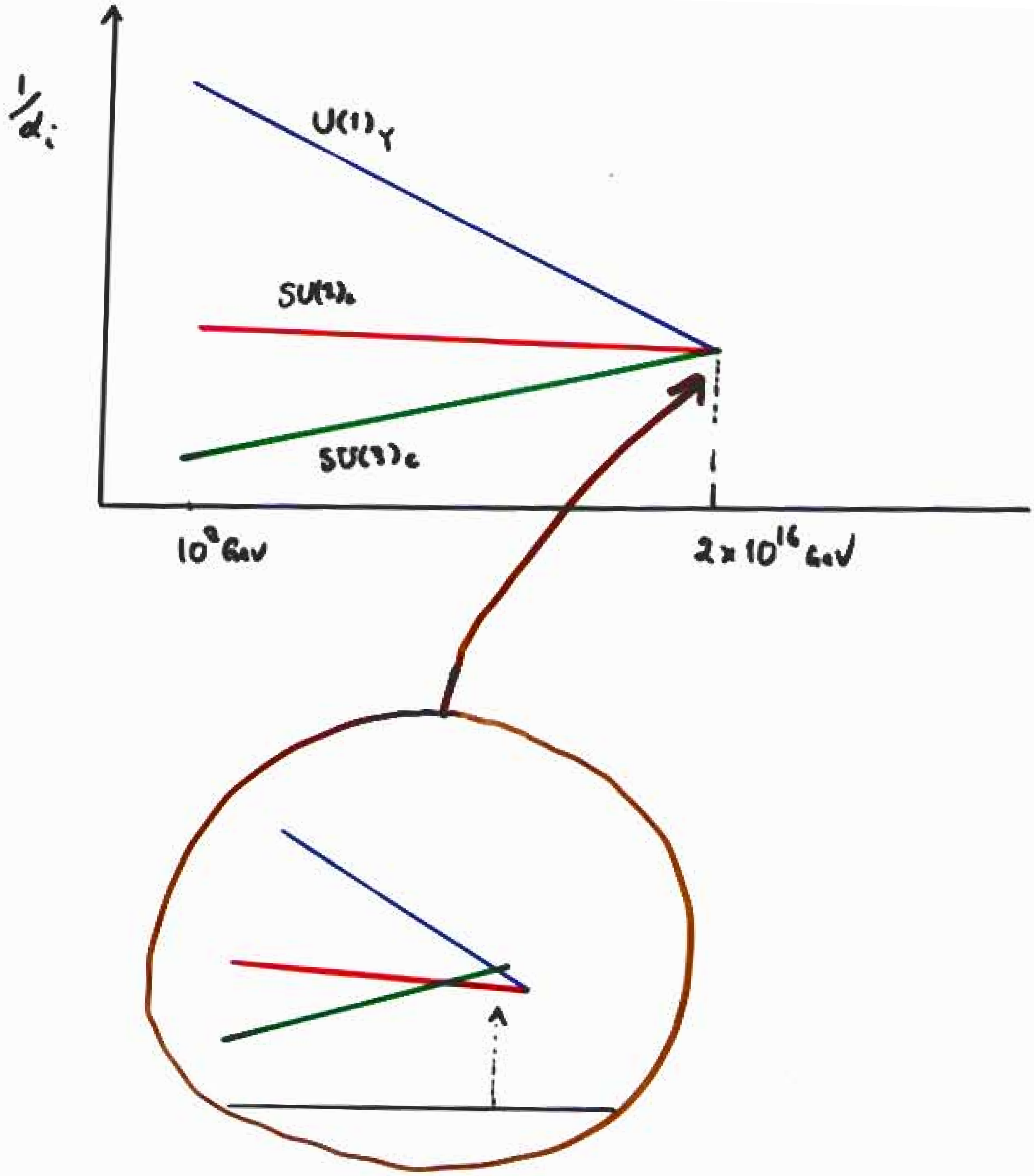
$$\frac{vol(\Sigma_1)}{(2\pi\sqrt{\alpha'})^4} \sim (10-100)$$

STRING SCALE $1/\sqrt{\alpha'} \sim 10^{17}$ GeV

Σ_1 : 4 CYCLE D7 WRAPPED AROUND

↓

MODERATELY LARGE VOLUME .



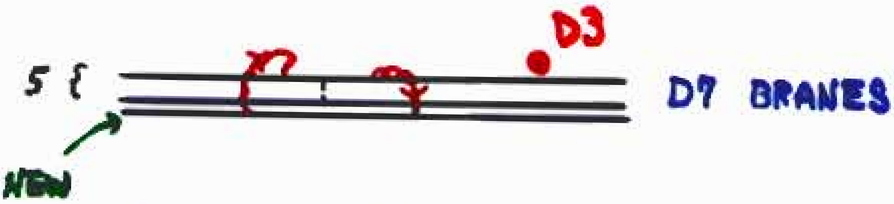
$$\frac{1}{\alpha_c}, \frac{1}{\alpha_Y} > \frac{1}{\alpha_{2L}}$$

$$\uparrow$$

$\frac{1}{\alpha_{3\,D3}}$ $\frac{1}{\alpha_{1\,D3}}$ CONTRIBUTIONS

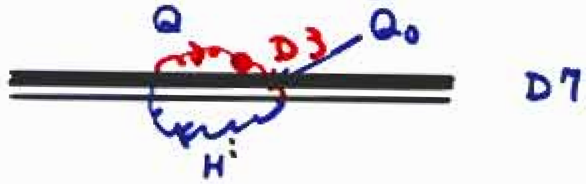
Fig 1

HIGGS MULTIPLETS : D7-D7 OPEN STRING



$$H^{i}(\mathbf{5}) + \bar{H}_{i}(\mathbf{5}^{*})$$

SUPERPOTENTIAL :



$$W = \lambda\, Q^{a}_{i} H^{i} \bar{Q}^{0}_{a} + \bar{\lambda}\, Q^{a}_{0} H_{i} \bar{Q}^{i}_{a}$$

$$\langle Q^{a}_{i} \rangle = v\, \delta^{a}_{i} \qquad \langle \bar{Q}^{i}_{a} \rangle = v\, \delta^{i}_{a}$$

$$\Rightarrow \lambda v\, H^{a}_{c} \bar{Q}^{0}_{a} + \bar{\lambda} v\, Q^{a}_{0} H^{c}_{a}$$

H^{a}_{c} , $\bar{H}^{c}_{a}$ HAVE GUT-SCALE MASSES.

H_{f} , $\bar{H}_{f}$ REMAIN MASSLESS !!!

: QUARKS AND LEPTONS :

IN $SU(5)_{GUT}$

$$\underline{10} = \left[\begin{pmatrix} u \\ d \end{pmatrix} \quad \bar{u} \quad \bar{e} \right]$$

$$\underline{5}^* = \left[\bar{d} \quad \begin{pmatrix} \nu \\ e \end{pmatrix} \right]$$

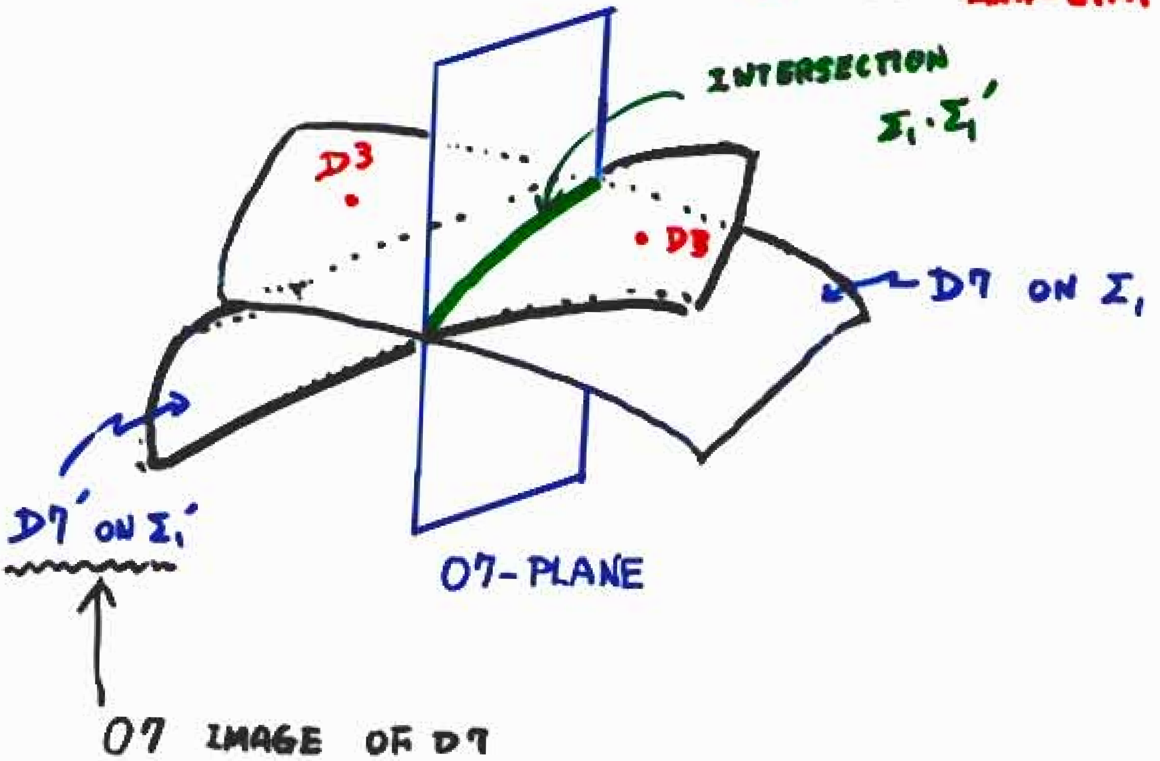
QUARK (DOUBLET) MIXING $\simeq$ MIXING OF $\underline{10}$'s

LEPTON (DOUBLET) MIXING $\simeq$ MIXING OF $\underline{5}^*$'s

WHY $\theta_{ij}(\underline{5}^*) \gg \theta_{ij}(\underline{10})$?

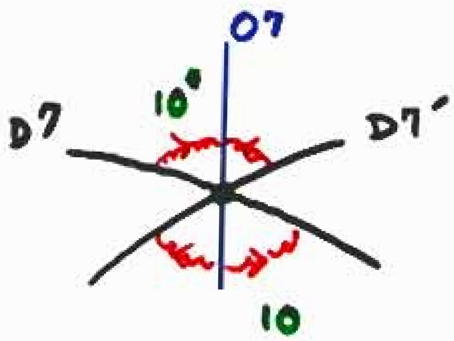
CONSTRUCTION OF $\underline{\mathbf{10}}$

$\underline{\mathbf{10}} = \begin{array}{|c|}\hline \ \\ \hline \ \\ \hline\end{array}$ ANTI-SYM.



$\left(\begin{array}{c}\underline{\mathbf{10}} \\ + \\ \underline{\mathbf{10}}^*\end{array}\right)$: D7 - D7$'$ OPEN STRINGS

LIVE ON $\Sigma_1 \cdot \Sigma_1'$ (2 DIMENSION)



WE HAVE $\underline{10} + \underline{10}^*$ IN (5+1) DIM. SPACETIME.

SUPPOSE NON-TRIVIAL BACKGROUND F_{45} OF $U(1)_5$.

$$U(1)_5 = \begin{pmatrix} 1 & & & & & 0 \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ 0 & & & & & -5 \end{pmatrix}$$

$$\int F_{45}\, dx^4 dx^5 = N$$

{ TOGETHER WITH COMPACTIFICATION }

THEN, WE HAVE N $\underline{10}$'s AND 0 $\underline{10}^*$.

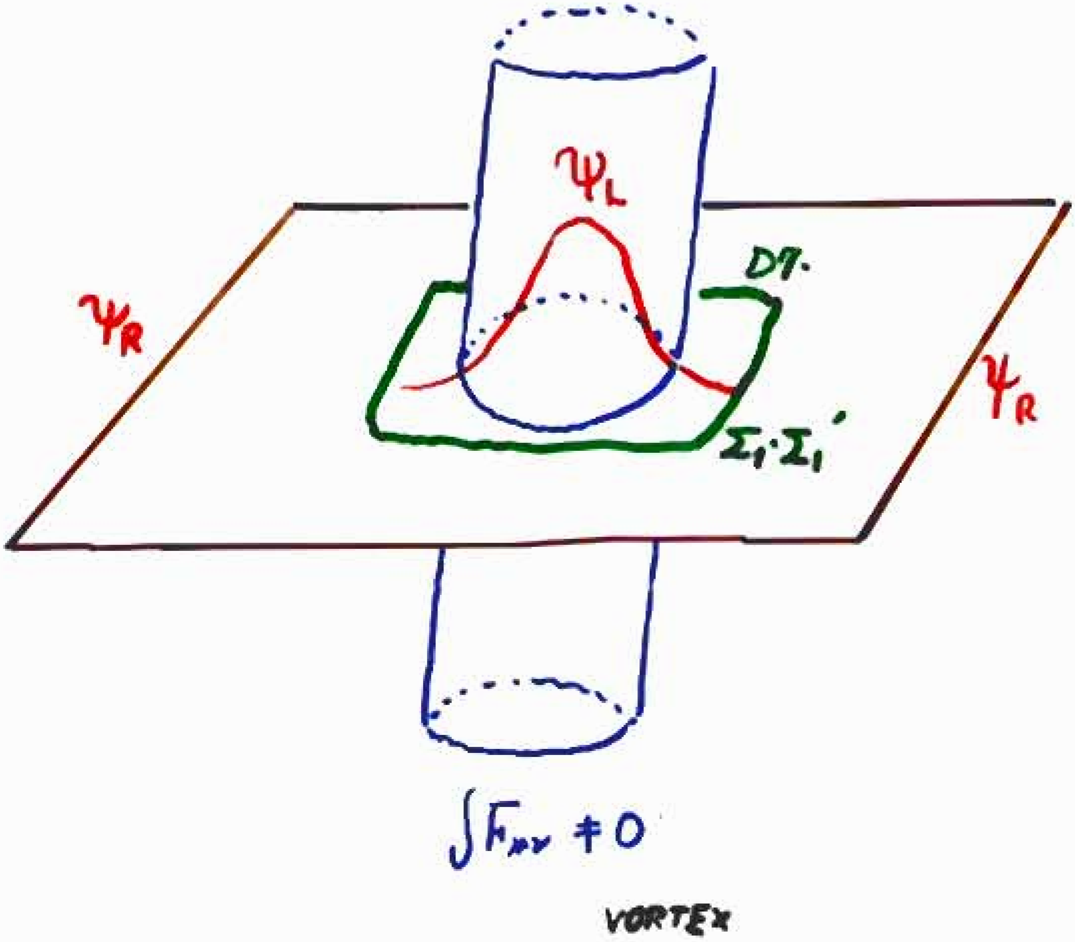
Fig.2

WE CHOOSE $N=3$. { 3 FAMILIES }

IN (3+1) DIM. SPACETIME

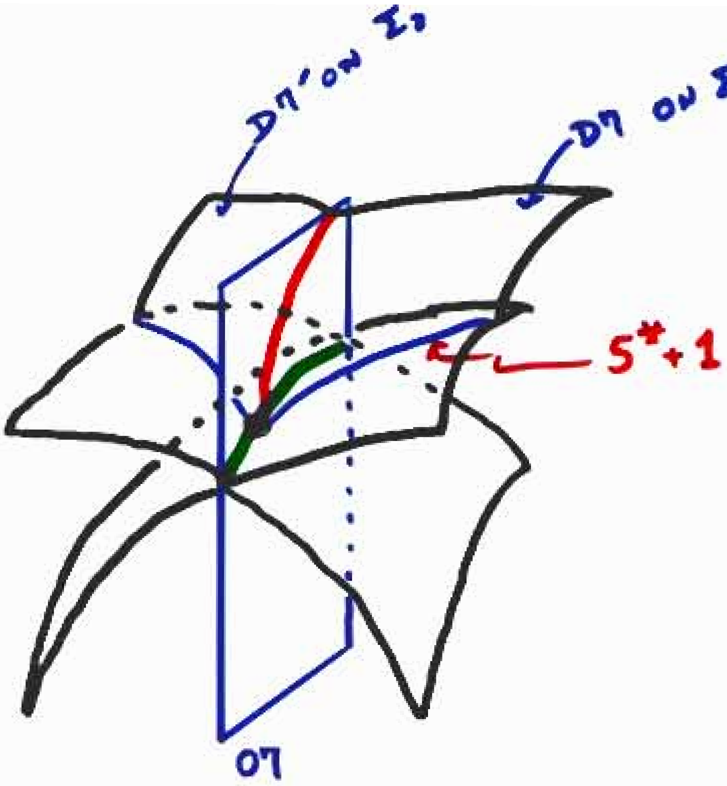
BUT, THEORY HAS ANOMALIES.

U(1) GAUGE THEORY WITH Ψ_L AND Ψ_R.

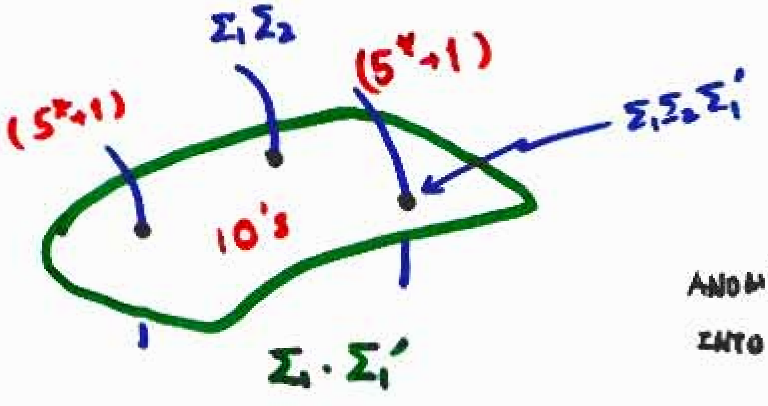


ON $\Sigma_1 \cdot \Sigma_1'$ WE HAVE A CHIRAL GAUGE THEORY WITH Ψ_L.

Fig 2.



(5+1) DIM. SPACETIME

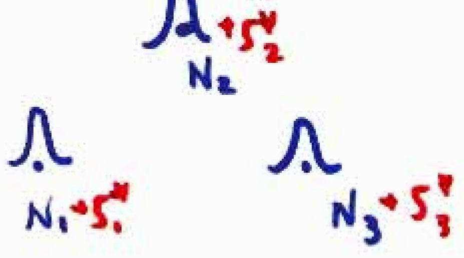


ANOMALIES INFLOW
INTO POINTS
$\Sigma_1 \Sigma_1' \Sigma_2$.

10's ARE IN 2 DIM. SPACE $(\Sigma_1 \cdot \Sigma_1')$.

$(5^* + 1)$'s ARE LOCALIZED AT 3 POINTS.

• MAJORANA MASS MATRIX FOR SINGLETS $N(1)$ IS NEARLY DIAGONAL UNLESS THE POINTS ARE CLOSE TO EACH OTHER.



• DIRAC YUKAWA MATRIX FOR $5^*_i N_j H$ IS NEARLY DIAGONAL :

$$(N_1 \cdot N_2 \cdot N_3) \begin{pmatrix} M_1 & & \varepsilon \\ & M_2 & \\ \varepsilon & & M_3 \end{pmatrix} \begin{pmatrix} N_1 \\ N_2 \\ N_1 \end{pmatrix}$$

$$(5^*_1,\ 5^*_2,\ 5^*_3) \begin{pmatrix} a & & \varepsilon \\ & b & \\ \varepsilon & & c \end{pmatrix} \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}$$

→ $\therefore$ NEUTRINO MASS MATRIX IS ALMOST DIAGONAL.

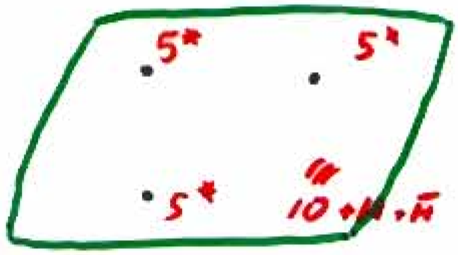
see saw

"LARGE NEUTRINO MIXINGS COME FROM CHARGED LEPTON MIXINGS."

Altarelli, Feruglio, Masina
hep-ph/0402155

$$U_{MNS} = U_e^\dagger U_\nu$$

$$\simeq U_e^\dagger$$



$5^* \times 10 \times H$ YUKAWA COUPLING

$\hookrightarrow$ CHARGED LEPTON MASS MATRIX

$$(5^*_1 \; 5^*_2 \; 5^*_3) \begin{pmatrix} a & b & c \\ a & b & c \\ a & b & c \end{pmatrix} \begin{pmatrix} 10_1 \\ 10_2 \\ 10_3 \end{pmatrix}$$

5^* ANARCHY

Hall, Murayama

$$U_{MNS} \simeq U_e^\dagger$$

$$\simeq \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{bmatrix}$$

5^* DEMOCRATIC

Fritzsch, Xing

Fukugita, Tanimoto

T.Y.

$$U_{MNS} \simeq \begin{bmatrix} * & * & \varepsilon \\ * & * & * \\ * & * & * \end{bmatrix}$$

BI-LARGE MIXINGS WITH $\theta_{13} \ll 1$.

$a \sim \varepsilon^2$, $b \sim \varepsilon$, $c \sim 1$

$$\rightarrow \quad m_e : m_\mu : m_\tau \simeq \varepsilon^2 : \varepsilon : 1$$

$$(10_1, 10_2, 10_3) \begin{pmatrix} \varepsilon^6 & \varepsilon^3 & \varepsilon^4 \\ \varepsilon^5 & \varepsilon^2 & \varepsilon \\ \varepsilon^3 & \varepsilon & 1 \end{pmatrix} \begin{pmatrix} 10_1 \\ 10_2 \\ 10_3 \end{pmatrix}$$

$$m_u : m_c : m_t \simeq \varepsilon^4 : \varepsilon^2 : 1$$

$$U_{CKM} \simeq \begin{bmatrix} 1 & \varepsilon & \varepsilon^2 \\ \varepsilon & 1 & \varepsilon \\ \varepsilon^3 & \varepsilon & 1 \end{bmatrix}$$

SMALL QUARK MIXINGS

CONCLUSION

1. PRODUCT-GROUP UNIFICATION SOLVES THE DOUBLET-TRIPLET SPLITING PROBLEM

2. IT IS NATURALLY EMBEDDED INTO A BRANE WORLD IN TYPE IIB STRING THEORY.

   Watari. T.Y. (04)

3. SUCH A EXTENSION LEADS US TO THE AFM HYPOTHESIS. (04) AND THE OBSERVED BI-LARGE MIXINGS OF NEUTRINOS ARE NATURALLY EXPLAINED.

PREDICTION

THE UNIFICATION SCALE IS LOWER THAN THE GUT SCALE.

THE PROTON DECAYS THROGH DIM.= 6 OPERATORS.

$\tau(p \to e^+ + \pi^0) \lesssim$ [redacted] $\times 10^{34}$ years

Ibe. Watari