A Natural Framework for Bi-large Neutrino Mixing

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Outline

- Frampton-Glashow-Yanagida ansatz

  Bi-large neutrino mixing

  Minimal CP violation

- Natural FGY in

  SUSY $\times$ [SU(2) $\times$ U(1)]$_{FS}$

  SUSY SO(10) $\times$ [SU(2) $\times$ U(1)^n]$_{FS}$

Neutrinos : Masses and Mixing Angles

- $\Delta m^2_{atm} = |m_3^2 - m_2^2| \approx 3 \times 10^{-3}\ \mathrm{eV}^2$
  $\sin 2\theta_{atm} \approx 1$
- $\Delta m^2_{sol} = |m_2^2 - m_1^2| \approx 7 \times 10^{-5}\ \mathrm{eV}^2$
  $\sin 2\theta_{sol} \leq 1$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \approx \begin{pmatrix} c_{sol} & s_{sol} & 0 \\ -s_{sol}/\sqrt{2} & c_{sol}/\sqrt{2} & 1/\sqrt{2} \\ -s_{sol}/\sqrt{2} & c_{sol}/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Neutrinos masses $\sim m_\nu^T\ M_N^{-1}\ m_\nu$

Large Mixing Angles

- in m_ν (Dirac mass) ?
- in M_N (Majorana mass) ?
- or in m_e (Charged lepton mass matrix) ?

Frampton-Glashow-Yanagida ansatz

$$\begin{aligned}\mathcal{L} &= \left(\nu_e\, a + \nu_\mu\, a'\, e^{-i\phi/2}\right)\, N_1 \\ &\quad + \left(\nu_\mu\, b + \nu_\tau\, b'\right)\, N_2 \\ &\quad + \tfrac{1}{2}\left(M_1\, N_1^2 + M_2\, N_2^2\right) \\ \mathcal{L} &\equiv \nu\, D^{tr}\, N + \tfrac{1}{2}\, N\, M_N\, N\end{aligned}$$

$b \approx b'$, $a \approx a'$ $\Longrightarrow$ Bi-Large mixing

$$\mathcal{M}_{FGY} = D^{tr}\; M_N^{-1}\; D$$

$$D^{tr} = \begin{pmatrix} a & 0 \\ a'\, e^{-i\phi/2} & b \\ 0 & b' \end{pmatrix}$$

$$m_{\nu_3} \approx 2b^2/M_2 \approx 0.05\ \text{eV} = \sqrt{\Delta m^2_{atm}}$$

$$m_{\nu_2} \approx 2a^2/M_1 \approx 8.4 \times 10^{-3}\ \text{eV} = \sqrt{\Delta m^2_{sol}}$$

$$m_{\nu_1} = 0 \quad \theta_{13} \sim m_{\nu_2}/(\sqrt{2}\, m_{\nu_3})$$

FGY – Raidal and Strumia analysis

Best fit to atmospheric and solar data $\Longrightarrow$

- $\theta_{13} = 0.078 \pm 0.015$ Observable
- $m_{ee}^{0\nu\beta\beta} = 2.6 \pm 0.4$ meV Unobservable

Successful leptogenesis – Two solutions

- $$M_1 \ll M_2 \quad M_1 \approx 10^{11}\ \mathrm{GeV}/|\sin\phi| \quad \Longrightarrow \quad \phi < 0$$
  $$P(\nu_e \to \nu_\mu) < P(\nu_\mu \to \nu_e) \equiv P(\bar{\nu}_e \to \bar{\nu}_\mu)$$
  $$SUSY \quad B(\mu \to e\gamma) \approx 2\ r\ 10^{-13}$$
  $$B(\tau \to \mu\gamma) \geq 3\ r\ 10^{-12}$$

- $$M_1 \gg M_2 \quad M_2 \approx 10^{12}\ \mathrm{GeV}/|\sin\phi| \quad \Longrightarrow \quad \phi > 0$$
  $$P(\nu_e \to \nu_\mu) > P(\nu_\mu \to \nu_e) \equiv P(\bar{\nu}_e \to \bar{\nu}_\mu)$$
  $$SUSY \quad B(\tau \to \mu\gamma) \approx 7\ r\ 10^{-11}$$
  $$B(\mu \to e\gamma) \geq r\ 10^{-11}$$

$$r \approx (\tan\beta/10)^2\ (150\ \mathrm{GeV}/m_{SUSY})^4$$

"Natural" FGY texture in SUSY w/ $[SU(2)\times U(1)]_{FS}$

Lepton weak doublets: $l_i = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}$

$L_a = \begin{pmatrix} l_1 \\ l_2 \end{pmatrix}$ - flavor doublet, $\quad l_3$ - flavor singlet

Superpotential:

$$W = \frac{H_u}{M} \left(L_a \, \phi^a \, N_1 + L_a \, \tilde{\phi}^a \, N_2 + l_3 \, \omega \, N_2 \right) + \frac{1}{2} \left(S_1 \, {N_1}^2 + S_2 \, {N_2}^2 \right)$$

VEVs $\langle \phi \rangle = \begin{pmatrix} \langle \phi^1 \rangle \\ \langle \phi^2 \rangle \end{pmatrix}$ and $\langle \tilde{\phi} \rangle = \begin{pmatrix} 0 \\ \langle \tilde{\phi}^2 \rangle \end{pmatrix}$

NO fine tuning

$\langle S_i \rangle = M_i, \;\; i = 1, 2 \qquad \langle H_u \rangle = \begin{pmatrix} 0 \\ v \sin\beta/\sqrt{2} \end{pmatrix}$

$$a = v \sin\beta \, \frac{\langle \phi^1 \rangle}{\sqrt{2} M}, \qquad a' \, e^{-i\phi/2} = v \sin\beta \, \frac{\langle \phi^2 \rangle}{\sqrt{2} M},$$

$$b = v \sin\beta \, \frac{\langle \tilde{\phi}^2 \rangle}{\sqrt{2} M}, \qquad b' = v \sin\beta \, \frac{\langle \omega \rangle}{\sqrt{2} M}$$

Charged lepton masses

$$W_{ch.\ leptons} = \frac{H_d}{M} \left(L_a\ \phi^a\ \bar{e}_1\ +\ L_a\ \tilde{\phi}^a\ \bar{e}_2\ + \right.$$
$$\left. l_3\ (\ \omega\ \bar{e}_2\ +\ \bar{\omega}\ \bar{e}_3\ ) \right)$$

$$\langle H_d \rangle = \begin{pmatrix} v \cos\beta/\sqrt{2} \\ 0 \end{pmatrix}$$

$$\bar{a} = v \cos\beta\ \frac{\langle \phi^1 \rangle}{\sqrt{2}M}, \qquad \bar{a}'\ e^{-i\phi/2} = v \cos\beta\ \frac{\langle \phi^2 \rangle}{\sqrt{2}M},$$
$$\bar{b} = v \cos\beta\ \frac{\langle \tilde{\phi}^2 \rangle}{\sqrt{2}M}, \qquad \bar{b}' = v \cos\beta\ \frac{\langle \omega \rangle}{\sqrt{2}M},$$
$$\bar{c} = v \cos\beta\ \frac{\langle \bar{\omega} \rangle}{\sqrt{2}M}$$

$$m_l = \begin{pmatrix} \bar{a} & \bar{a}'\ e^{-i\phi/2} & 0 \\ 0 & \bar{b} & \bar{b}' \\ 0 & 0 & \bar{c} \end{pmatrix}$$

Charged lepton masses – continued

$$m_l = \begin{pmatrix} \bar{a} & \bar{a}'\ e^{-i\phi/2} & 0 \\ 0 & \bar{b} & \bar{b}' \\ 0 & 0 & \bar{c} \end{pmatrix}$$

- $\bar{a},\ \bar{a}' \ll \bar{b},\ \bar{b}' \ll \bar{c}$
- $m_e \approx \bar{a}, \quad m_\mu \approx \bar{b}, \quad m_\tau \approx \bar{c}$

$m_l^{diagonal} = U_e^\dagger\ m_l\ U_e \quad U_e \approx Diag(1, e^{i\phi/2}, -e^{i\phi/2})$

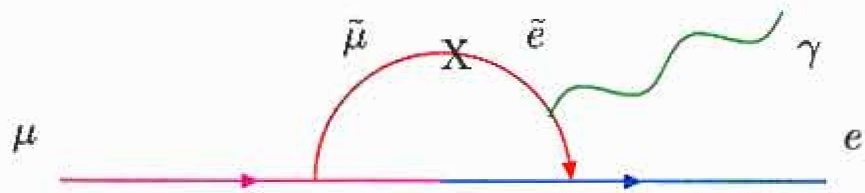
$$\mathcal{M} = \ U_e^{tr}\ [\ D^{tr}\ M_N^{-1}\ D\ ]\ U_e \approx \mathcal{M}_{FGY}$$

$$D^{tr} = \begin{pmatrix} a & 0 \\ a'\ e^{-i\phi/2} & b \\ 0 & b' \end{pmatrix}$$

- $(m_e/m_\mu)^2 \approx (\bar{a}/\bar{b})^2 \approx (a/b)^2 \approx (M_1/M_2)(m_{\nu_2}/m_{\nu_3})$
- $\Longrightarrow (M_1/M_2) \sim 10^{-4}$ [RS Ambiguity Resolved !]

SU_2 family symmetry
Ameliorates SUSY Flavor Problem

— Dine, Kagan & Leigh; Pomarol & Tomassini; Barbieri, Dvali & Hall



What Scale ?
$M \sim M_1 \sim M_2 \gg M_Z$

Leptogenesis $\Longrightarrow$ $M_1,\ M_2\ >\ 10^{11}$ GeV

$M\ =\ M_{GUT}$?

SUSY GUTs

- $M_{GUT} \sim 3 \times 10^{16}$ GeV
- $M_Z << M_{GUT}$ "Natural"
- Explains Charge Quantization
- Predicts Gauge Coupling Unification*
- Predicts Yukawa Coupling Unification
- + Family Symmetry $\Longrightarrow$

  Hierarchy of Fermion Masses

  Protects against large flavor violation
- Neutrino Masses via See - Saw scale $\sim 10^{-3} - 10^{-2}\ M_G$

  $\sim M_G^2/M_{Pl}$
- LSP – Dark Matter Candidate
- Baryogenesis via Leptogenesis

SO_{10} SUSY GUT $\times$ $[SU_2 \times U_1^n]_{FS}$

— Barbieri, Hall, S.R. & Romanino; Blazek, S.R. & Tobe

The superpotential for the charged fermion sector, including the heavy Froggatt-Nielsen states — $\{ \chi^a, \ \bar{\chi}_a \}$

$$W \supset \quad 16_3 \; 10_H \; 16_3 + 16_a \; 10_H \; \chi^a + \bar{\chi}_a \left(M_\chi \, \chi^a + \; 45 \, \frac{\phi^a}{M} \, 16_3 \; + \; 45 \, \frac{\tilde{\phi}^a \tilde{\phi}^b}{M^2} \, 16_b + A^{ab} \, 16_b \right)$$

where

- $16_a \quad a, b = 1, 2$; 16_3 — 3 families
- $10_H \Longrightarrow$ Higgs doublets
- $M_\chi = M(1 \; + \; \alpha \, X \; + \; \beta \, Y)$

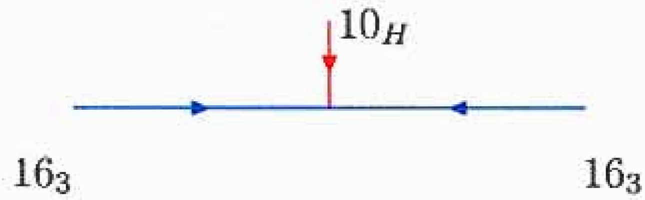
X, Y — SO(10) breaking vevs in the adjoint representation

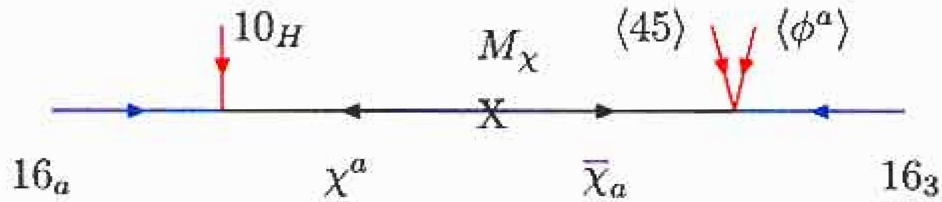
X corresponding to the U(1) in SO(10) which preserves SU(5),

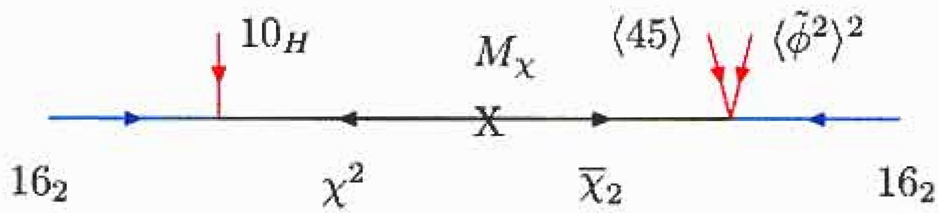
Y is standard weak hypercharge and

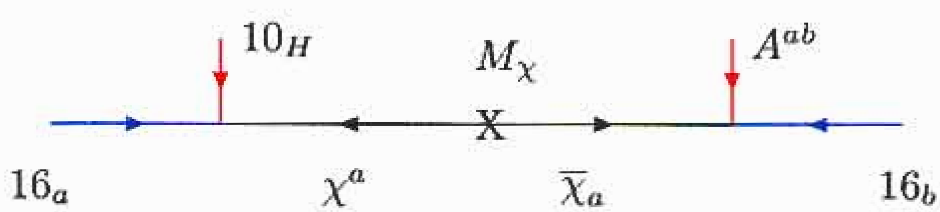
- α, β are arbitrary parameters.

Effective Fermion Mass Operators









Effective Yukawa Couplings

$$Y_u = \begin{pmatrix} 0 & \epsilon' \rho & -\epsilon \xi \\ -\epsilon' \rho & \tilde{\epsilon} \rho & -\epsilon \\ \epsilon \xi & \epsilon & 1 \end{pmatrix} \lambda$$

$$Y_d = \begin{pmatrix} 0 & \epsilon' & -\epsilon \xi \sigma \\ -\epsilon' & \tilde{\epsilon} & -\epsilon \sigma \\ \epsilon \xi & \epsilon & 1 \end{pmatrix} \lambda \qquad (1)$$

$$Y_e = \begin{pmatrix} 0 & -\epsilon' & 3 \epsilon \xi \\ \epsilon' & 3 \tilde{\epsilon} & 3 \epsilon \\ -3 \epsilon \xi \sigma & -3 \epsilon \sigma & 1 \end{pmatrix} \lambda$$

with

$$\xi = \langle \phi^1 \rangle / \langle \phi^2 \rangle; \quad \tilde{\epsilon} \propto (\langle \tilde{\phi}^2 \rangle / M)^2; \qquad (2)$$

$$\epsilon \propto \langle \phi^2 \rangle / M; \quad \epsilon' \sim \langle A^{12} \rangle / M;$$

$$\sigma = \frac{1 + \alpha}{1 - 3\alpha}; \quad \rho \sim \beta \ll \alpha.$$

Features of the Model

1. Family Hierarchy

$$SU_2 \times U_1 \longrightarrow U_1 \longrightarrow \text{nothing}$$

$$\epsilon,\ \tilde{\epsilon} \qquad \epsilon'$$

3rd family >> 2nd family >> 1st family

2. Patterns - approximate Georgi - Jarlskog "natural"

$$\langle 45 \rangle = (B - L) M_G$$

$$m_s \sim \tfrac{1}{3} m_\mu$$

$$m_d \sim 3 m_e$$

3. $\lambda_t = \lambda_b = \lambda_\tau = \lambda_{\nu_\tau} = \lambda \ @M_G$

4. $\beta \ll \alpha \sim 1 \implies m_u < m_d$ even though $m_t >> m_b$

5. Gauge Coupling Unification

6. SU_2 suppresses flavor violation such as $\mu \to e\gamma$

7. 10 Yukawa parameters fits 13 fermion masses and mixing angles

following *Blazek, S.R. & Tobe*

Observable	Data(σ) (masses	*Theory* in GeV)	*Pull*
M_Z	91.188 (0.091)	91.21	< 0.50
M_W	80.419 (0.080)	80.40	< 0.50
$G_\mu \cdot 10^5$	1.1664 (0.0012)	1.166	< 0.50
α_{EM}^{-1}	137.04 (0.14)	137.0	< 0.50
$\alpha_s(M_Z)$	0.11720 (0.002)	0.1139	2.65
M_t	174.30 (5.1)	171.3	< 0.50
$m_b(M_b)$	4.220 (0.09)	4.377	3.04
$M_b - M_c$	3.400 (0.2)	3.430	< 0.50
$m_c(m_c)$	1.3000 (0.15)	1.212	< 0.50
m_s	0.089 (0.011)	0.100	1.01
m_d/m_s	0.050 (0.015)	0.0751	2.80
Q^{-2}	0.00203 (0.00020)	0.00200	< 0.50
M_τ	1.777 (0.0018)	1.777	< 0.50
M_μ	0.10566 (0.00011)	.1057	< 0.50
$M_e \cdot 10^3$	0.5110 (0.00051)	0.5110	< 0.50
V_{us}	0.2230 (0.0040)	0.2213	< 0.50
V_{cb}	0.04020 (0.0019)	0.0391	< 0.50
V_{ub}/V_{cb}	0.0860 (0.008)	0.0850	< 0.50
V_{td}	0.00820 (0.00082)	0.00846	< 0.50
ϵ_K	0.00228 (0.00023)	0.00233	< 0.50
$\sin 2\beta$	0.7270 (0.036)	0.6898	1.07
$B(b \to s\gamma) \cdot 10^4$	3.340 (0.38)	3.433	< 0.50
TOTAL χ^2	12.16		

Bi-large Neutrino Mixing in $SO_{10} \times [SU_2 \times U_1^n]_{FS}$ model

$$Y_\nu = \begin{pmatrix} 0 & -\epsilon' \, \omega & \frac{3}{2}\, \epsilon \, \xi \, \omega \\ \epsilon' \, \omega & 3 \, \tilde{\epsilon} \, \omega & \frac{3}{2} \, \epsilon \, \omega \\ -3 \, \epsilon \, \xi \, \sigma & -3 \, \epsilon \, \sigma & 1 \end{pmatrix} \lambda$$

with $\omega \; = \; 2\,\sigma/(2\,\sigma - 1)$

$\Longrightarrow$ Dirac neutrino mass matrix – $m_\nu \equiv Y_\nu \frac{v}{\sqrt{2}} \sin\beta$

$$W_{neutrino} = \; \frac{\overline{16}}{M} \left(N_1 \, \tilde{\phi}^a \; 16_a \; + \; N_2 \, \phi^a \; 16_a \; + \; N_3 \, \theta \; 16_3 \right) + \frac{1}{2} \left(S_1 \; {N_1}^2 \; + \; S_2 \; {N_2}^2 \right)$$

- Effective neutrino mass

$$W_\nu^{eff} = \; \nu \; m_\nu \; \bar{\nu} + \bar{\nu} \; V \; N + \frac{1}{2} \; N \; M_N \; N$$

$$(V^{tr})^{-1} = \frac{M}{v_{16}} \begin{pmatrix} -1/(\langle\tilde{\phi}^2\rangle\,\xi) & 1/\langle\phi^1\rangle & 0 \\ 1/\langle\tilde{\phi}^2\rangle & 0 & 0 \\ 0 & 0 & 1/\langle\theta\rangle \end{pmatrix}$$

- $$\mathcal{M} = U_e^{tr}\,[\, m_\nu\,(V^{tr})^{-1}\, M_N\, V^{-1}\, m_\nu^{tr}\,]\, U_e$$

$$D^{tr} \equiv m_\nu\,(V^{tr})^{-1}\, M_N \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} a & 0 \\ a' & b \\ 0 & b' \end{pmatrix}$$

- $$\mathcal{M} = U_e^{tr}\,[\, D^{tr}\,\hat{M}_N^{-1}\, D\,]\, U_e$$

$$\hat{M}_N \equiv \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}$$

Gives "natural" Glashow, Frampton & Yanagida ansatz

$\Longrightarrow$ Bi-large neutrino mixing matrix

$$Y_\nu \; (V^{tr})^{-1} \sim$$

$$\begin{pmatrix} 0 & -\epsilon'\,\omega & \frac{3}{2}\,\epsilon\,\xi\,\omega \\ \epsilon'\,\omega & 3\,\tilde{\epsilon}\,\omega & \frac{3}{2}\,\epsilon\,\omega \\ -3\,\epsilon\,\xi\,\sigma & -3\,\epsilon\,\sigma & 1 \end{pmatrix}$$

$$\times \begin{pmatrix} -1/(\langle\tilde{\phi}^2\rangle\,\xi) & 1/\langle\phi^1\rangle & 0 \\ 1/\langle\tilde{\phi}^2\rangle & 0 & 0 \\ 0 & 0 & 1/\langle\theta\rangle \end{pmatrix}$$

$$\sim \begin{pmatrix} a & 0 & \mathrm{X} \\ a' & b & \mathrm{X} \\ 0 & b' & \mathrm{X} \end{pmatrix}$$

- ZEROs are EXACT !!

$b \sim b'$: "Natural"

$$\epsilon' \sim \epsilon\, \xi$$

$$b \equiv \quad \epsilon'\, \omega\, \lambda\, (M_2/\phi^1)\, \frac{M}{v_{16}} \frac{v \sin\beta}{\sqrt{2}}$$

$$b' \equiv \quad -3\, \epsilon\, \xi\, \sigma\, \lambda\, (M_2/\phi^1)\, \frac{M}{v_{16}} \frac{v \sin\beta}{\sqrt{2}}.$$

$a \sim a'$: minor fine-tuning O(1/10)

$$\epsilon'\, \xi^{-1} \sim \tilde{\epsilon} \Longrightarrow \text{minor fine-tuning}$$

$$a \equiv \quad -\epsilon'\, \omega\, \lambda\, (M_1/\tilde{\phi}^2)\, \frac{M}{v_{16}} \frac{v \sin\beta}{\sqrt{2}}$$

$$a' \equiv \quad (-\epsilon'\, \xi^{-1} + 3\, \tilde{\epsilon})\, \omega\, \lambda\, (M_1/\tilde{\phi}^2)\, \frac{M}{v_{16}} \frac{v \sin\beta}{\sqrt{2}}$$

$M_1/M_2 \sim 10^3$

$$m_{\nu_2}/m_{\nu_3} \approx (m_e/m_\mu)\, (M_1/M_2)\, \tilde{\epsilon}$$

Summary

- Frampton-Glashow-Yanagida ansatz
  - One CP violating angle $\Longrightarrow$ Sign of matter - anti-matter asymmetry correlated with the CP violating neutrino oscillations.

    EXCEPT for Raidal-Strumia ambiguity ($M_1 \ll M_2$ or $M_1 \gg M_2$)!
  - Natural framework for bi-large neutrino mixing !
- SUSY $\times$ $[SU(2) \times U(1)]_{FS}$
  - FGY ansatz "naturally" and resolves the RS ambiguity.
- SUSY SO(10) $\times$ $[SU(2) \times U(1)^n]_{FS}$
  - FGY ansatz "naturally" and bi-large neutrino mixing is "natural."
  - But, in general, there are more CP violating angles.