Neutrino Masses, Unification And Our Origin

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I. Introduction

A) Why Neutrinos are special ?

Since the discoveries/Confirmations of Atmospheric & Solar ν- oscillations, ν's have emerged as being among the most Effective probes into the nature of Higher Unification

→ Simply because of their non-Zero but tiny masses

$$m_{\nu_e}/m_e \lesssim 10^{-6} \; ; \; \boxed{(m_{\nu_\tau}/m_{top}) \sim 10^{-11}}$$

→ A Subtle clue, via the seesaw mechanism, to some of the deepest Laws of Nature pertaining to

- The Unification - Scale
- Nature of the Unification - Symmetry

→ In this Sense, ν's provide us with a rare window to view physics at truly short distances ($\sim 10^{-30}$ cm).

→ Furthermore, it seems most likely that tiny ν- masses are also at the root of the

→ Origin of matter - Anti-matter Asymmetry

→ Thus ν's may also be crucial to our own origin !

IB) Why ν-Masses Suggest Physics Beyond SM?

SM: $SU(2)_L \times U(1)_Y \times SU(3)^c$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \; u_R, \; d_R, \; \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \; e_R, \; \cancel{\nu}_R$$

ν_L can acquire Majorana Mass Using Quantum gravity :

$$\lambda_L \, LLHH / M_{Planck} \; + hc \; (|\Delta L| = 2)$$

$$\rightarrow m(\nu_L) \sim \lambda_L \, (250 \, GeV)^2 / 10^{19} \, GeV$$

$$\sim \lambda_L \, (0.6) \times 10^{-5} \, eV \quad \text{(Too Small)}$$

$$\updownarrow$$

SuperK $\Rightarrow \sqrt{\Delta m^2_{23}} \approx 1/20 \, eV$

Can argue Need New Physics at an eff. Scale $\sim 10^{15}$ GeV $\longleftrightarrow$ Can Link this to Scale of meeting of 3 gauge Couplings $\sim 2 \times 10^{16}$ GeV

$\rightarrow$ HINT AT A LINK BETWEEN ν-OSCILL & Grand Unification !

IC) Present A Unified Picture of

- fermion Masses & Mixings
- ν - Oscillations
- CP & Flavor Violations
- Baryogenesis via Leptogenesis

Main Theme will be to exhibit how all these pieces & more fit together neatly within a single predictive Unified Picture based on Symmetry $G(224) = SU(2)_L \times SU(2)_R \times SU(4)^c$ or $SO(10)$ in 4D with SUSY.

Crucial Ingredients of this picture are:

- EXISTENCE OF $\nu_R \longleftrightarrow SU(4)^c$ or $SU(2)_L \times SU(2)_R$
- SUSY UNIF. SCALE $\longleftrightarrow$ SUSY GUT // $\boxed{M(\nu_R)}$
- Symmetry $SU(4)^c \left\{ \begin{array}{l} \longleftrightarrow m_b^0 \approx m_\tau \text{ \& More} \\ \longleftrightarrow \boxed{m(\nu^\tau_{Dirac}) \approx m_t(M_X)} \\ \longleftrightarrow \text{B-L protects } M(\nu_R) \end{array} \right.$
- Seesaw $\longleftrightarrow \boxed{m(\nu_L^\tau) \approx m(\nu^\tau_{Dirac})^2 / M(\nu_R)}$

Success of the Unified Framework, involving 8 predictions for fermion masses & ν-oscillations, and for leptogenesis

→ Strong Evidence in favor of the 4 ingredients:
ν_R / SUSY Unif / $SU(4)^c$ / See Saw

$\Downarrow$

Minimally An Eff. Symmetry
String $SU(2)_L \times SU(2)_R \times SU(4)^c$ // $SO(10)$ // in 4D.

Plan

II) Symmetries Beyond the SM

III) Insight From SuperK Result: $\boxed{\sqrt{\Delta m^2_{23}} \sim \frac{1}{20}\,\text{eV}}$

III (q, l, ν) Masses & Mixings Within a Predictive G(224) or SO(10) - Framework (BPW)

IV ν Masses ⟷ Leptogenesis " (Pati, 2003)

V ν Masses ⟷ C̸P, LFV, " (B, P, Rastogi 2004)

VI ν Masses ⟷ Proton Decay " (BPW)

VII Summary

II) Symmetries Beyond the SM : Family Multiplet Structure

$$G(213) = SU(2)_L \times U(1)_{Y_W} \times SU(3)^c$$

$$\begin{pmatrix} u_r & u_y & u_b \\ d_r & d_y & d_b \end{pmatrix}_L^{1/3} ; \ (u_{r,y,b})_R^{4/3} ; \ (d_{r,y,b})_R^{-2/3} , \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L^{-1} ; (e^-_R)^{-2}$$

$$Q_{em} = I_{3L} + Y_W/2$$

5 disconnected multiplets in 1 Family // Y_W ? //

Q_{em}? // $Q_{e^-} = -Q_p$ // Co-exist of (q, ℓ) // g_1, g_2, g_3?

$$\Downarrow$$

P, S 72/73

$$\boxed{G(224) = [SU(2)_L \times SU(2)_R \times SU(4)^c] \otimes (L \leftrightarrow R)}$$

$$F^e_{L,R} = \begin{bmatrix} u_r & u_y & u_b & \nu_e \\ d_r & d_y & d_b & e^- \end{bmatrix}_{L,R}$$

All 16 in one L-R conj. mult / Y_W / Q_{em} quantized / $Q_{e^-} = -Q_p$ / $\{q, \ell\}$ unif / (W, E, S) / $\boxed{\nu_R}$ / $\boxed{B-L}$ Generator

$$\boxed{Q_{em} = I_{3L} + I_{3R} + \frac{B-L}{2}}$$

$$\Downarrow$$

$SO(10)$: $\underset{\sim}{16}$ (1 gauge coupling / $g_1 = g_2 = g_3$ at M_U)

(Georgi // Fritzsch, Minkowski (74/75)

$G(213) \to SU(5)$

$\bar{5} + 10$

Georgi & Glashow (74)

NO ν_R, NO B-L

III) Insight From SuperK Result: $\sqrt{\Delta m^2_{23}} \approx 1/20$ eV

SeeSaw

$$\boxed{m(\nu_L^\tau) \approx \frac{m(\nu^\tau_{\text{Dirac}})^2}{M(\nu_R^\tau)}}$$

Gell-Mann, Ramond, Slansky // Yanagida // Mohapatra, Senjanovic

Ignore Mixing for a moment

(a) $\boxed{m(\nu^\tau_{\text{Dirac}}) \simeq m_t(M_X) \approx 120\,\text{GeV}} \Leftarrow$ SU(4) - Color

~~SU(5)~~, ~~[SU(3)]~~^{3}

$\boxed{m_b^0 \approx m_\tau^0}$?

(b) Get $M(\nu_R^\tau)$ from SUSY Unif. Scale: $M_X \approx 2\times10^{16}$ GeV

$$\underset{(\approx 1)}{f_{33}} \frac{16_3\, 16_3 \langle \overline{16}_H \rangle \langle \overline{16}_H \rangle}{M \to \sim 10^{18}\,\text{GeV}} \Rightarrow M(\nu_R^\tau) \sim \frac{(2\times10^{16}\,\text{GeV})^2}{10^{18}\,\text{GeV}}$$

$$\approx \boxed{4\times10^{14}\,\text{GeV}\,(1/2 - 2)}$$

$$\Downarrow$$

$$\boxed{m(\nu_L^\tau) \sim \frac{(120\,\text{GeV})^2}{4\times10^{14}\,\text{GeV}} \approx \left(\frac{1}{30}\,\text{eV}\right)\left(\frac{1}{2}\text{ to }2\right)}$$

Also get $m(\nu_L^\mu) \sim \frac{m(\nu_L^\tau)}{10} \Rightarrow \sqrt{\Delta m^2(\nu_2\nu_3)}_{Th} \approx \left(\frac{1}{30}\,\text{eV}\right)\left(\frac{1}{2} - 2\right)$

Thus SuperK result brings to light the existence of ν_R // reinforces the ideas of a) SeeSaw // (b) SU(4) Color // & (c) SUSY Unif.

IV (q, l, ν) - Masses & Mixings Within a Predictive G(224) or SO(10)-Framework (Babu, Pati Wilczek)

Minimal Higgs For SO(10) - Breaking

$\boxed{45_H, 16_H, \overline{16}_H, 10_H}$; $\underbrace{\cancel{126}_H, \cancel{120}_H}, \cancel{54}_H$

Allowed by String Solns

Too large GUT-Scale Threshold Corrections // Seems disallowed by strings

$$\begin{tikzcd} SO(10) \arrow[d, "\langle 16_H\rangle \sim M_{GUT}", "\substack{\langle 45_H\rangle \sim \\ \propto B-L}"'] \\ SU(2)_L \times U(1)_Y \times SU(3)^c \arrow[d, "\approx v_{EW}", "\langle 10_H\rangle"'] \\ U(1)_{em} \times SU(3)^c \end{tikzcd}$$

- Only allowed Cubic Coupling $16_i\, 16_j\, 10_H \rightarrow$ B-L indep // Symmetric in $i \leftrightarrow j$
- Need B-L dependence $\rightarrow m_s^0 \sim m_\mu^0/3$ & Also Antisymmetric piece

Assume Flavor Symmetries⊛ Such that

- 3rd Family gets the dominant contribution → $h_{33}\, 16_3\, 16_3\, 10_H$
  The lighter Families get their masses primarily Through off-diagonal mixings with the heavier families :

"33" ≫ "23" ≫ "22" ≫ "12" ≫ "11" etc.

	16_3	16_2	16_1	10_H	16_H	$\overline{16}_H$	45_H	S
U(1)→	a	$a+1$	$a+2$	$-2a$	$-a-1/2$	$-a$	0	-1

$$h_{33}\; 16_3\, 16_3\, 10_H + \tilde{h}_{23}\; 16_2\, 16_3\, 10_H \overset{\boxed{\sim 1/10}}{\boxed{(S/M)}}$$

$$+ \tilde{h}_{22}\, 16_2\, 16_2\, 10_H \langle S/M \rangle^2 + \cdots\cdots$$

"a" gets fixed from LMA Soln → $\nu_L^e \nu_L^\mu$ Mass

$16_1 16_2 16_H 16_H 10_H 10_H / M^3$ Leading if $\boxed{a = -1/2}$

⊛ String Solns generically do yield Flavor Symmetries

A Concrete Example: Minimal Higgs: $\{45_H, 16_H, \overline{16}_H, 10_H\}$ + "S"

1st/ consider Only μ & τ Families (Flavor sym: $\mu \neq \tau$)

$$\mathcal{L}_{mass} = h_{33}\, 16_3\, 16_3 \langle 10_H \rangle \rightarrow \text{3rd Family: } \underline{m_b^0 = m_\tau^0}$$

$$\propto ① \leftarrow h_{33}$$

$$\propto \textcircled{\sigma} \leftarrow + \tilde{h}_{23}\, 16_2\, 16_3 \langle 10_H \rangle \langle S/M \rangle \rightarrow \text{2nd Family}$$

$$\propto \textcircled{\epsilon} \leftarrow + a_{23}\, 16_2\, 16_3 \underbrace{\langle 10_H \rangle \overset{\text{ANTISYMMETRIC}, \propto B-L}{\langle 45_H \rangle}}_{M} \rightarrow m_\mu^0 \neq m_s^0$$

$$\propto \textcircled{\hat{\eta}} \leftarrow + g_{23}\, 16_2\, 16_3 \underline{\langle 16_H \rangle} \langle \overline{16}_H^D \rangle_{EW} / M$$

$\hookrightarrow$ CKM $\neq 1$

$$10_H \rightarrow (2 \leftrightarrow 2) \ll (2 \leftrightarrow 3) \ll (3,3) \text{ Flavor}_{\text{sym}}$$

$$U = \begin{array}{cc} c & t \\ \left(\begin{matrix} 0 & \epsilon+\sigma \\ -\epsilon+\sigma & 1 \end{matrix}\right) \end{array} m_U^0 \qquad D = \begin{array}{cc} s & b \\ \left(\begin{matrix} 0 & \epsilon+\eta \\ -\epsilon+\eta & 1 \end{matrix}\right) \end{array} m_D^0$$

$$N_{Dirac} = \begin{array}{cc} \nu_\mu & \nu_\tau \\ \left(\begin{matrix} 0 & -3\epsilon+\sigma \\ +3\epsilon+\sigma & 1 \end{matrix}\right) \end{array} m_U^0 \qquad L = \begin{array}{cc} \mu & \tau \\ \left(\begin{matrix} 0 & -3\epsilon+\eta \\ 3\epsilon+\eta & 1 \end{matrix}\right) \end{array} m_D^0$$

Note $q-\ell$ correlation (SU(4)-color)

up-down " $(SU(2)_L \times SU(2)_R)$

$$\boxed{\eta \equiv \hat{\eta} + \sigma}$$

Dirac Masses (3 Families)

$$U = \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & 0 & \epsilon+\sigma \\ 0 & -\epsilon+\sigma & 1 \end{pmatrix} m_U^0 \; ; \; D = \begin{pmatrix} 0 & \epsilon'+\eta' & 0 \\ -\epsilon'+\eta' & 0 & \epsilon+\eta \\ 0 & \epsilon+\eta & 1 \end{pmatrix} m_D^0$$

$$N = \begin{pmatrix} 0 & -3\epsilon' & 0 \\ 3\epsilon' & 0 & -3\epsilon+\sigma \\ 0 & 3\epsilon+\sigma & 1 \end{pmatrix} m_U^0 \; ; \; L = \begin{pmatrix} 0 & \epsilon'+\eta' & 0 \\ -\epsilon'+\eta' & 0 & -3\epsilon+\eta \\ 0 & +3\epsilon+\eta & 1 \end{pmatrix} \times m_D^0$$

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2 New param (ϵ', η'), but 5 new observables just in (q, ℓ) System $\Rightarrow$ 3 New predictions for (q,ℓ) // With $\epsilon' = 0$ $\to m_u \to 0$

ν Majorana Masses

Saw before, $M_R \approx 10^{15}$ GeV

Expect $y \sim 1/10$

$$M_R^\nu = \begin{pmatrix} x & 0 & z \\ 0 & 0 & y \\ z & y & 1 \end{pmatrix} M_R$$

Note Same hier. pattern as in Dirac Sector

$$f_{ij}\, 16_i\, 16_j \langle \overline{16}_H \rangle \langle \overline{16}_H \rangle / M_{st}$$

$$\downarrow$$

$$f_{ij}\, \nu_{iR}^T\, C^{-1} \nu_{jR} \, \langle \overline{16}_H \rangle^2 / M_{st}$$

$$(M_R^\nu)_{ij} = f_{ij} \langle \overline{16}_H \rangle^2 / M_{st}$$

Including $m_U^0 \to$ 7 param $(\eta, \epsilon, \sigma, \eta', \epsilon', m_U^0, m_D^0)$

describing $9 \times 4 = 36$ entries → Will it work ?

Input: Assume all param real for a moment

$m_t^{phys} = 174$ GeV ; $m_c(m_c) = 1.37$ GeV,
$m_s(1\,\text{GeV}) = 116$ MeV, $m_\mu, m_\tau, m_u(M_X) = 1.5$ MeV, m_e

$\Downarrow$

$\sigma \simeq 0.110, \eta \simeq 0.151, \epsilon \simeq -0.095,$
$\epsilon' = \sqrt{m_u/m_c}\,(m_c/m_t) \simeq 2 \times 10^{-4}$; $\eta' \simeq \sqrt{m_e/m_\mu}\left(\frac{m_c}{m_t}\right) \simeq 4 \times 10^{-3}$
$m_U^0 = m_t(M_X) \simeq 110$ GeV ; $m_D^0 \simeq 1.5$ GeV

Majorana Mass of ν_R's : $f_{ij}\, 16_i\, 16_j\, \overline{16}_H\, \overline{16}_H / M$

$$\Rightarrow f_{ij} (\nu_R^{iT} C^{-1} \nu_R^j) \langle 16_H \rangle^2 / M$$

$$M_R^\nu = \begin{pmatrix} x & 0 & z \\ 0 & 0 & y \\ z & y & 1 \end{pmatrix} \boxed{M_R}$$

determined by $m_{\nu_2}/m_{\nu_3} \approx 1/7$

calculated $\approx 5 \times 10^{14}$ GeV.

6 New observables.

Summary on Fermion Masses & Mixings (BPW)

Predictions	Observations
$m_b(m_b) \approx (4.7-4.9)\,\text{GeV}$	$\approx 4.2\,\text{GeV}$
$m(\nu_\tau) \approx (1/30\,\text{eV})\left(\frac{1}{2}-2\right)$	$\approx (1/15 - 1/25)\,\text{eV}$ ⊛
$V_{cb} \simeq 0.042$ ↕	≈ 0.04 ↕
$\sin^2 2\Theta^{osc}_{\nu_\mu\nu_\tau} \simeq 0.92 \leftrightarrow \underset{\text{LMA}}{\boxed{0.99}}$	$\approx 0.92 \leftrightarrow 1$
$V_{us} \approx 0.22$	≈ 0.21
$\lvert V_{ub}\rvert \simeq 0.0032$	$\approx 0.003 - 0.004$
$m_d(1\,\text{GeV}) \simeq 8\,\text{MeV}$	$\approx 8-10\,\text{MeV}$

$$m(\nu_\mu) \approx (2-10)\times10^{-3}\,\text{eV} \longleftrightarrow \begin{cases} \text{SMA} \sim 3\times10^{-3}\,\text{eV} \\ \boxed{\text{LMA} \simeq 7\times10^{-3}\,\text{eV}} \end{cases}$$

$$m(\nu_e) \sim (1 \text{ to few})\times10^{-3}\,\text{eV}$$

↕ Consistent with the framework

$$M(\nu_R^\tau, \nu_R^\mu, \boxed{\nu_R^e}) \approx \left(10^{15}, 2\times10^{12}, (1/3-3)\times10^{10}\right)\ \text{GeV}$$

Just right for Lepto/Baryogenesis

ng -| pa

Predictions

Writing only for 2×2 (for simplicity)

$$U = \begin{array}{cc} c & t \\ \left(\begin{matrix} 0 & \epsilon+\sigma \\ -\epsilon+\sigma & 1 \end{matrix}\right. & \left.\right) \end{array} m_U^0 \qquad D = \begin{array}{cc} s & b \\ \left(\begin{matrix} 0 & \epsilon+\eta \\ -\epsilon+\eta & 1 \end{matrix}\right. & \left.\right) \end{array} m_D^0$$

$$N = \begin{pmatrix} 0 & -3\epsilon+\sigma \\ 3\epsilon+\sigma & 1 \end{pmatrix} m_U^0 \qquad L = \begin{array}{cc} \mu & \tau \\ \left(\begin{matrix} 0 & -3\epsilon+\eta \\ 3\epsilon+\eta & 1 \end{matrix}\right. & \left.\right) \end{array} m_D^0$$

$$\Downarrow$$

$$m_b^0 \approx m_\tau^0 (1-8\epsilon^2) \Rightarrow \boxed{m_b(m_b) \simeq 4.7 \text{ GeV}}$$

$$V_{cb} = \left| \underbrace{\sqrt{\frac{m_s}{m_b}}}_{(0.156)} \underbrace{\left(\frac{\eta+\epsilon}{\eta-\epsilon}\right)^{1/2}}_{(1/2.2)} - \sqrt{\frac{m_c}{m_t}} \left(\frac{\sigma+\epsilon}{\sigma-\epsilon}\right)^{1/2} \right| = |\sigma-\eta|$$

$$\boxed{=0.042}$$

Suppressed

ENHANCED ≃ 1.8

$$\theta^{osc}_{\nu_\mu \nu_\tau} = |\theta^l_{\mu\tau} - \theta^\nu_{\mu\tau}| \approx \left| \sqrt{\frac{m_\mu}{m_\tau}} \left(\frac{\eta-3\epsilon}{\eta+3\epsilon}\right)^{1/2} + \sqrt{\frac{m_{\nu_2}}{m_{\nu_3}}} \right|$$

$$= 0.437 + \sqrt{m_{\nu_2}/m_{\nu_3}} \quad \boxed{\approx 0.3}$$

Expt

$$\Rightarrow \boxed{\sin^2 2\theta^{osc}_{\nu_\mu \nu_\tau} = 0.92 \leftrightarrow 0.99} \qquad .92 \leftrightarrow 1$$

$$m_{\nu_2}/m_{\nu_3} = 1/15 \longleftrightarrow 1/7$$ LMA.

SMA or LMA ?

$\rightarrow f_{ij}\, 16_i\, 16_j\, \overline{16}_H\, \overline{16}_H / M$

Just with Standard See-saw ν_L-masses, SMA rather generic

$$m(\nu_L^e) \sim 2\times10^{-5} - 2\times10^{-6}\,\text{eV} \;//\; m(\nu_L^\mu) \sim 3\times10^{-3}\,\text{eV}$$

$$\Theta^{osc}_{\nu_e\nu_\mu} = \Theta^{\ell}_{e\mu} - \Theta^{\nu}_{e\mu} \approx 0.05$$

Situation alters once allow for direct Maj. masses of ν_L's — Most likely to arise through Higher Dim. op. involving GUT & EW VEV's - Through tiny $\sim 10^{-3}$eV entries → IMPORTANT For (ν_e-ν_μ)

$$W \supset g_{12}\, \underbrace{16_1\, 16_2}\, \underset{\downarrow}{16_H}\, \underset{\downarrow}{16_H}\, \underbrace{10_H\, 10_H} / M^3_{GUT}$$

$$g_{12}\,(\nu^e_L\, \nu^\mu_L)\,(\langle\tilde{\nu}_{RH}\rangle/M_{GUT})^2\, \underset{(\sim 175\,\text{GeV})^2}{(v_u^2)}$$

$$\boxed{\sim g_{12}(\nu^e_L\, \nu^\mu_L)\,(1.5-6)\times10^{-3}\text{eV}} \quad (\langle 16_H\rangle \approx (1-2)\, M_{GUT})$$

$$\begin{array}{cc} \nu_L^e & \nu_L^\mu \\ \end{array}$$
$$\begin{bmatrix} \approx 0 & (3-4) \\ (3-4) & 7 \end{bmatrix} \times10^{-3}\text{eV} \Longrightarrow \begin{bmatrix} \Theta^{\nu}_{\nu_e\nu_\mu} \approx 1/2 \\ \sin^2 2\Theta^{osc}_{\nu_e\nu_\mu} \sim 0.7 \end{bmatrix} \text{ Quite Plausible}$$

Thus LMA not strictly a prediction, but perfectly plausible within the framework.

Θ_{13}

$$m(\nu_L^e \nu_L^\tau)_{\text{Non-seesaw}} \sim (2-6)\times 10^{-3}\,\text{eV}$$

$$\Theta_{13} \sim \frac{(2-6)\times 10^{-3}\,\text{eV}}{5\times 10^{-2}\,\text{eV}}$$

$$\sim \cdot 03 - \cdot 1$$

ν-Less 2β decay : $\Delta L = \pm 2$

$$m_{ee} = \left| \sum_i m_i U_{ei}^2 \right|$$

$m_1 \sim$ few $\times 10^{-3}$ eV , $m_2 \approx (6-8)\times 10^{-3}$ eV

$m_3 \approx 5\times 10^{-2}$ eV

$\Theta_{12} \approx 1/2$, $\Theta_{13} \sim 0.03 - 0.1$

$$\Downarrow$$

$$m_{ee} \sim (1 \text{ to } 6)\times 10^{-3}\,\text{eV}$$

Summary on Fermion Masses & Mixings in the G(224)/SO(10) Framework

Given the bizarre pattern of masses & mixings of quarks, charged leptons and neutrinos, it seems remarkable that the simple pattern of fermion mass matrices*, motivated in large part by the group th of G(224)/SO(10) and the assumption of minimality of the Higgs system makes 7 predictions in agreement with observation.

→ Study Proton Decay // Leptogenesis // $\not{C}\not{P}$ within this framework.

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* Need to understand the origin of flavor symmetries.
→ Hierarchical entries.

V ν Masses $\leftrightarrow$ Leptogenesis Within The Same G(224) // SO(10) - Framework

JCP (hep-ph/2002, Phy Rev 2003)

Idea: (Fukugita, Yanagida // Sphaleron → Kuzmin Rubakov & Shaposhnikov)

Inflation ⟶ Reheat ⟶ Superheavy ν_R's (N_1) From Thermal Bath // Or Non-Thermal Inflaton Decay

$\nu_R \to l + H$ & $\bar{l} + \bar{H}$ (& SUSY Analogs)

LEPTON ASYM Parameter

$$\boxed{\varepsilon_1 = \frac{1}{8\pi v^2 (M_D^\dagger M_D)_{11}} \sum_{j=2,3} \mathrm{Im}\left[(M_D^\dagger M_D)_{j1}\right]^2 f\left(M_j^2/M_1^2\right)}$$

$M_D = M^{\nu}_{Dirac}$ in a basis where M^{ν}_{R} is diagonal

$$\rightarrow \boxed{\varepsilon_1 \approx \left(\frac{1}{8\pi}\right)\left(\frac{m_t^0}{v_{EW}}\right)^2 \left|(\sigma + 3\epsilon) - y\right|^2 \underbracket{(\sin 2\phi_{21})}(-3)\left(\frac{M_1}{M_2}\right)}$$

ϕ_{21} = Eff. Phase From (Entries in) Dirac & Majorana Neutrino Mass Matrices

Note ε_1 depends crucially on Both M^{ν}_{Dirac} & $M^{(\nu_R)}_{Majorana}$.

$$\boxed{Y_L \equiv \frac{n_L - n_{\bar{L}}}{s} = \kappa\, \epsilon_1/g^*}$$ $(g^*_{MSSM} \approx 228)$

Wash out Factor } $\kappa \approx 10^{-4} \times (\tilde{m}_1/eV) \rightarrow$ Thermal (Büchmuller et al // Giudice et al)

$\approx 1 \rightarrow$ Non-Thermal

$$\tilde{m}_1 \equiv (m_D^\dagger m_D)_{11}/M_1 \sim (8-3)\times 10^{-3}\,eV$$

Lepton Excess $\xrightarrow[\text{Sphalerons}]{\text{EW}}$ $\boxed{Y_B = C\,Y_L \approx -Y_L/3}$

$\Downarrow$

$$\left[Y_B/\sin 2\Phi_{21}\right]_{Theory}$$

$\xrightarrow[T_{RH}\sim \text{few}\times 10^{9}\,GeV]{\text{THERMAL}}$ $(4-15)\times 10^{-11}$

$\xrightarrow[T_{RH}\sim 10^{8}\,GeV]{\text{Non-Thermal}}$ $(30\times 10^{-11})(1\text{ to }4)$

Goes Very Well with

$$(Y_B)_{WMAP} \approx (8.7 \pm 0.4)\times 10^{-11} \quad !$$

for $\boxed{\sin 2\Phi_{21}} \approx 1/2$ (Thermal) // $\approx (1/4$ to $1/16)$ (Non-Thermal)

in accord with Gravitino Constraint.

→ A Unified Description of fermion Masses, ν-Oscillation & Baryogenesis (via leptogenesis) within a single predictive Framewo

V $\not{CP}$ Within the Same G(224)/SO(10) Framework

Babu, JCP, Parul Rastogi (To appear)

Challenge: Preserving success of fermion masses & mixings, ν-oscillations, can observed CP & Flavor Violations emerge?

$$\boxed{\Delta m_K,\ \epsilon_K,\ \Delta m_{B_d},\ a(B_d \to J/\psi\, K_S)}$$

In addition $(edm)_{e,n}$, $\mu \to e\gamma$, $\tau \to \mu\gamma$ . .

→ Non-trivial Constraint

Find natural phases (10-30)% can preserve all the successes for fermion masses & neutrino osc with

$$\boxed{\eta_W \approx 0.3 \ \& \ \rho_W \approx 0.17} \longleftrightarrow$$

Very close To SM CKM

$\eta_W \approx 0.33$, $\rho_W \approx 0.19$

And Thus (agree with all Four

with SUSY Amp ~(5-10)% [illegible] SM amp.

Without this will need 3 miracles!

SUSY effects will show in

1) edm, 2) $\mu \to e\gamma$, $\tau \to \mu\gamma$.

VI CP̸ Within The Same G(224)// SO(10)-Framework

Babu, JCP, Parul Rastogi (To appear)

Assume SUSY Breaking Preserves Flavor & Chiralit at say $M_{st} \gtrsim M_{GUT}$ (Gaugino Med // msugra // ··· CMSSM, ··)

CP̸ & Flavor in $\boxed{SM \oplus SUSY}$ $\longrightarrow$ Yukawa // RGE // Phases in Fermion Mass Matrices

EXPTL. FACTS } Δm_K, ϵ_K, Δm_{B_d}, $a(B_d \to J/\psi K_s)$ In accord with SM CKM To within ~20%

Also limits on $\mu \to e\gamma$, $\tau \to \mu\gamma$, $(edm)_{e,n}$ etc.

Challenge: Can observed CP & Flavor Viols emerge preserving successes of the framework in fermion masses & ν- oscillations?

→ A Non-trivial challenge

Find → For natural phases (~10-30%) in Dirac Mass Parameters ($\sigma, \eta, \epsilon, \epsilon'$ ··) can yield obs. CP & flavor Viols., while preserving the successes in fermion masses & ν oscillations!

For phases in a natural range, get

$$\boxed{\hat{\eta}_W \approx 0.3\,,\ \hat{\rho}_W \approx 0.16} \longleftrightarrow \boxed{\begin{array}{c}\text{SM CKM}\\ \eta_W \approx 0.33\\ \rho_W \approx 0.19\end{array}}$$

close to SM CKM - Values

$\hookrightarrow$ $(\hat{SM} + SUSY)$ in accord with all 4 entities

with A(SUSY) typically ~ (1 to 10%) $\hat{A}_{SM}$, but A(SUSY) Important for ϵ_K //

$$\underline{\tilde{q}_i \times \tilde{q}_j} \longrightarrow \boxed{\begin{array}{l}\text{Calculate (Re, Im) } \delta^{ij}_{LL} \text{ // RR}\\ m_{\tilde{q}} \approx 1\,\text{TeV},\ x = 0.6\end{array}}$$

$$(\Delta m_K)_{Tot} \approx (\Delta m_K)_{\hat{SM}} \approx 3.4 \times 10^{-15}\,\text{GeV} \Big\} \frac{\text{Re}\,A(\text{SUSY})}{\text{Re}\,A(\hat{SM})} = -0.6\,\%$$

$$\boxed{(\epsilon_K)_{Tot} \approx \left(\frac{2}{3}\right)(\epsilon_K)_{\hat{SM}} \approx 2.1 \times 10^{-3}} \quad \boxed{\frac{\text{Im}(A_{SUSY})}{\text{Im}(A_{\hat{SM}})} \approx -33\%}$$

$$(\Delta m_{B_d})_{Tot} \approx (\Delta m_{B_d})_{\hat{SM}} \approx 3.4 \times 10^{-13}\,\text{GeV} \quad \left(\frac{SUSY}{\hat{SM}}\right)_{Re} \approx 0.5\%$$

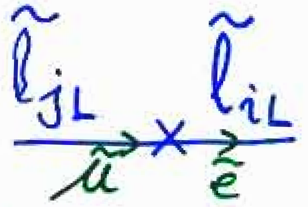
$$a(B_d \to J/\psi K_s)_{Tot} \approx 0.66 \qquad (SUSY/\hat{SM})_{Im} \approx +6\%$$

$$a(B_d \to \phi K_s)_{Tot} \approx 0.65$$

(Although δ^{23}_{RR} enhanced $\leftrightarrow \theta^{\nu}_{23}$, not enough to give Large deviation)

Thus the G(224)/SO(10) - Framework has met the Challenge so far as regards $C\!\!\!/P$, & quark - flavor Viol.

VI B) ν Masses ⟷ LFV



$$(\delta m^2_{\tilde{l}})_{ij} \approx -\frac{1}{8\pi^2}(3m_0^2 + a_0^2)(Y_\nu^\dagger Y_\nu)_{ij} \ln\frac{M}{M_{N_i}}$$

Dirac ν Mass → Basis Y_l & $Y(\nu_R)$ Diagonal

$$B(\mu \to e\gamma) = C\,(\alpha^3/G_F^2)\,\frac{|(\delta m^2_{\tilde{e}})_{12}|^2}{m_S^8}\tan^2\beta$$

$O(1)$ → can be $(1/10 - 5)$

$\boxed{\tan\beta = 3}$

$\boxed{(m_0, m_{1/2}, \mu, A_0)}$

$$\boxed{B(\mu \to e\gamma) \approx 10^{-12} - 10^{-14}}\ \begin{pmatrix}\text{BPW}\\ \text{BPR}\end{pmatrix} \to \begin{matrix}\text{Expt}\\ < 1.2\times 10^{-11}\end{matrix}$$

Can distinguish between SO(10)-Models

BPW

$$(M^D_\nu) \propto \begin{array}{cc} \nu_{\mu R} & \nu_{\tau R} \\ \left[\begin{matrix} 0 & \sigma - 3\epsilon \\ \sigma + 3\epsilon & 1 \end{matrix}\right. & \left.\right] \end{array}$$

$$M_l \propto \begin{array}{cc} \mu_R & \tau_R \\ \left[\begin{matrix} 0 & \eta - 3\epsilon \\ \eta + 3\epsilon & 1 \end{matrix}\right. & \left.\right] \end{array}$$

Albright & Barr (Lopsided)

$$M^D_\nu \propto \begin{bmatrix} 0 & \tilde{\epsilon} \\ -\tilde{\epsilon} & 1 \end{bmatrix}$$

$$M_l \propto \begin{bmatrix} 0 & \tilde{\sigma} + \tilde{\epsilon} \\ -\tilde{\epsilon} & 1 \end{bmatrix}$$

$\tilde{\sigma} \approx 1$

$\Rightarrow (Y_\nu)_{23}/(Y_\nu)_{33} = \sigma - 3\epsilon \longrightarrow \tilde{\epsilon}$

$(Y_l)_{23}/(Y_l)_{33} = \eta - 3\epsilon \longrightarrow \tilde{\sigma} + \tilde{\epsilon}$

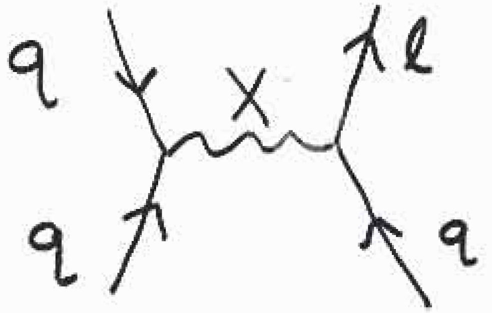
Diff $= \underset{.15}{\eta} - \underset{.11}{\sigma} \approx 0.04 \longrightarrow \approx 1$

$$\boxed{B(\mu \to e\gamma)_{\text{Albright, Barr}}/B(\mu \to e\gamma)_{\text{BPW}} \approx 625}$$

VII Link Between ν Masses & Proton Decay

In addition to

① Familiar d = 6 gauge boson Mediation

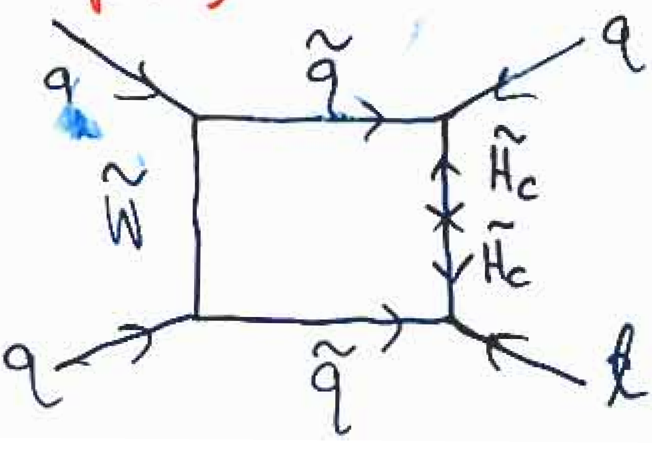


$\Gamma^{-1}(p \to e^+ \pi^0)$
$\approx 10^{35 \pm 1}$ yrs.

② Standard d = 5 Color-triplet Higgsino Mediated

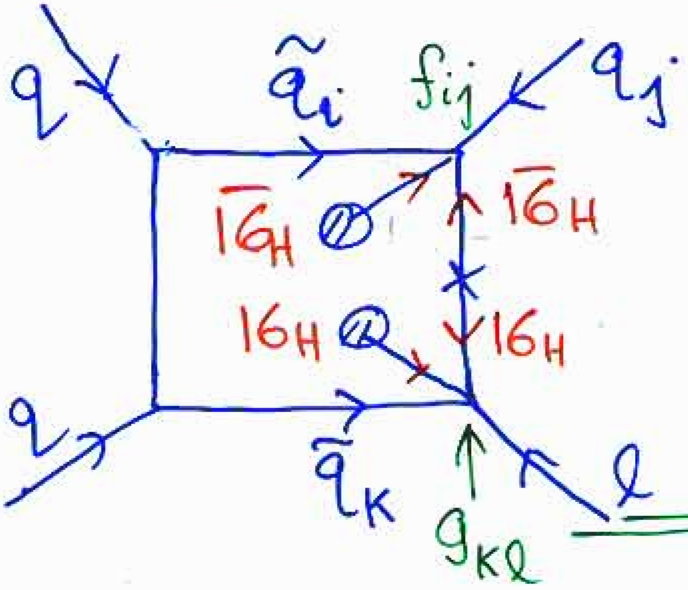
$$10_H = (2,2,1)_H + (1,1,\underset{\downarrow}{6})_H$$

Needs doublet-triplet splitting ← $\underset{\tilde{H}_c}{3^c} + \underset{\tilde{H}'_c}{3^{*c}}$



$\Gamma^{-1}(p \to \bar{\nu} K^+)$
$\lesssim 2 \times 10^{34}$ yrs.

③ New d=5 operators ↔ Majorana Masses of ν_R's



$$\Updownarrow$$

$f_{ij}\, 16_i\, 16_j\, \overline{16}_H\, \overline{16}_H / M$

↓

Generically $16_i \overline{16}_H$ in 45 & 1 of SO(10)

$\boxed{\text{ENTERS INTO DIRAC MASS MATRICES} \leftrightarrow V_{CKM} \neq \mathbb{1}}$

Babu, Pati & Wilczek (1997)

$\Downarrow$

$$\boxed{\Gamma^{-1}(p \to \bar{\nu} K^+)_{\text{New } d=5} \approx 10^{33} - 10^{34} \text{ yrs.}}$$

$\Downarrow$

$$\boxed{BR(p \to \mu^+ K^0)_{\text{New } d=5} \approx (10-30)\%}$$

SUSY SO(10) or G(224)

Note that these contribs would generically be present, even if "standard" d=5 operators med. by color triplets $\subset 10_H$ are absent, as in SUSY G(224)

The μ^+K^0 mode A Signature of this Mechanism

Summary on proton Decay : d=5

SUSY SU(5)/MSSM $\}$ $\boxed{\Gamma^{-1}(p\to\bar{\nu}K^{+}) \leq 2\times10^{31}\ \text{yrs}}$ Excluded by SuperK // IMB

SUSY SO(10)/MSSM $\}$ $\boxed{\Gamma^{-1}(p\to\bar{\nu}K^{+}) \lesssim 1.9\times10^{33}\ \text{yrs}}$ Tightly Constrained (Std. d=5)

SUSY SO(10)/ESSM $\}$ $\boxed{\Gamma^{-1}(p\to\bar{\nu}K^{+}) \lesssim 2\times10^{34}\ \text{yrs}}$ Std. d=5

SUSY G(224)/SO(10)/MSSM or ESSM $\}$ $\boxed{\Gamma^{-1}(p\to\bar{\nu}K^{+}) \approx 10^{33}-10^{34}\ \text{yrs}}$ New $d=5 \updownarrow$ (ν Masses)

Last Two fully Compatible with present limits, Must be seen with improvement by factor of 5-10, or else SUSY SO(10)/ or String G(224), which is otherwise so successful, would have a major set back.

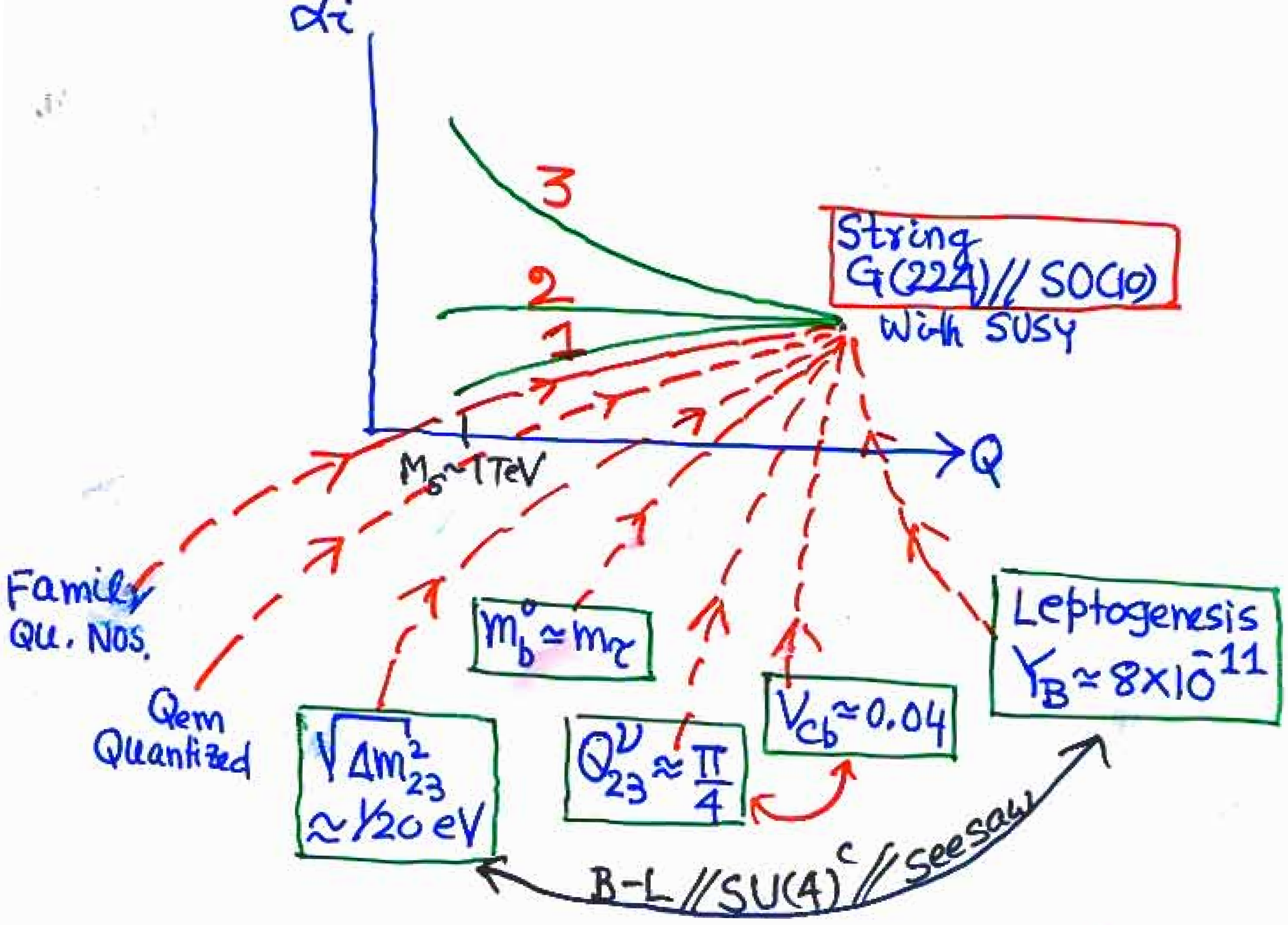
proton decay : d=6 : SUSY SU(5)/SO(10)//

and Flipped SU(5)

Higher Dim. GUTS →

$$\boxed{\Gamma^{-1}(p\to e^{+}\pi^{0}) \approx 10^{35.3\pm1.3}\ \text{yrs}}$$

VIII Rendevouz In Particle Physics



All these features hang together neatly within a Single Unified framework → Hard to believe this can be a mere Coincidence.

Rendevous Incomplete → Two Missing Links

1) Supersymmetry → LHC

2) Proton Decay → Need Next Gen. Detector.

Conclusion

The tiny mass of the neutrino ($m_\nu < 1\mathrm{eV}$) holds the key to:

(i) THE UNIFICATION SCALE
$M_X \approx 2\times10^{16}$ GeV
dist $\approx 10^{-30}$ cm
$\rightarrow M_R^{\nu} \sim 10^{15}$ GeV

(ii) NATURE OF THE UNIFICATION SYMMETRY
SU(4) - Color
$m(\nu^\tau)_{Dirac} = m_{top}(M_X)$
$\rightarrow$ G(224)/SO(10) Route, ~~SU(5)~~

(iii) ORIGIN OF AN EXCESS OF MATTER OVER ANTIMATTER
$\rightarrow$ OUR OWN ORIGIN!

The SeeSaw Mechanism enables us to turn the key!

Proton Decay and Supersymmetry remain The Two Missing Links.