KEK, 23 February '04

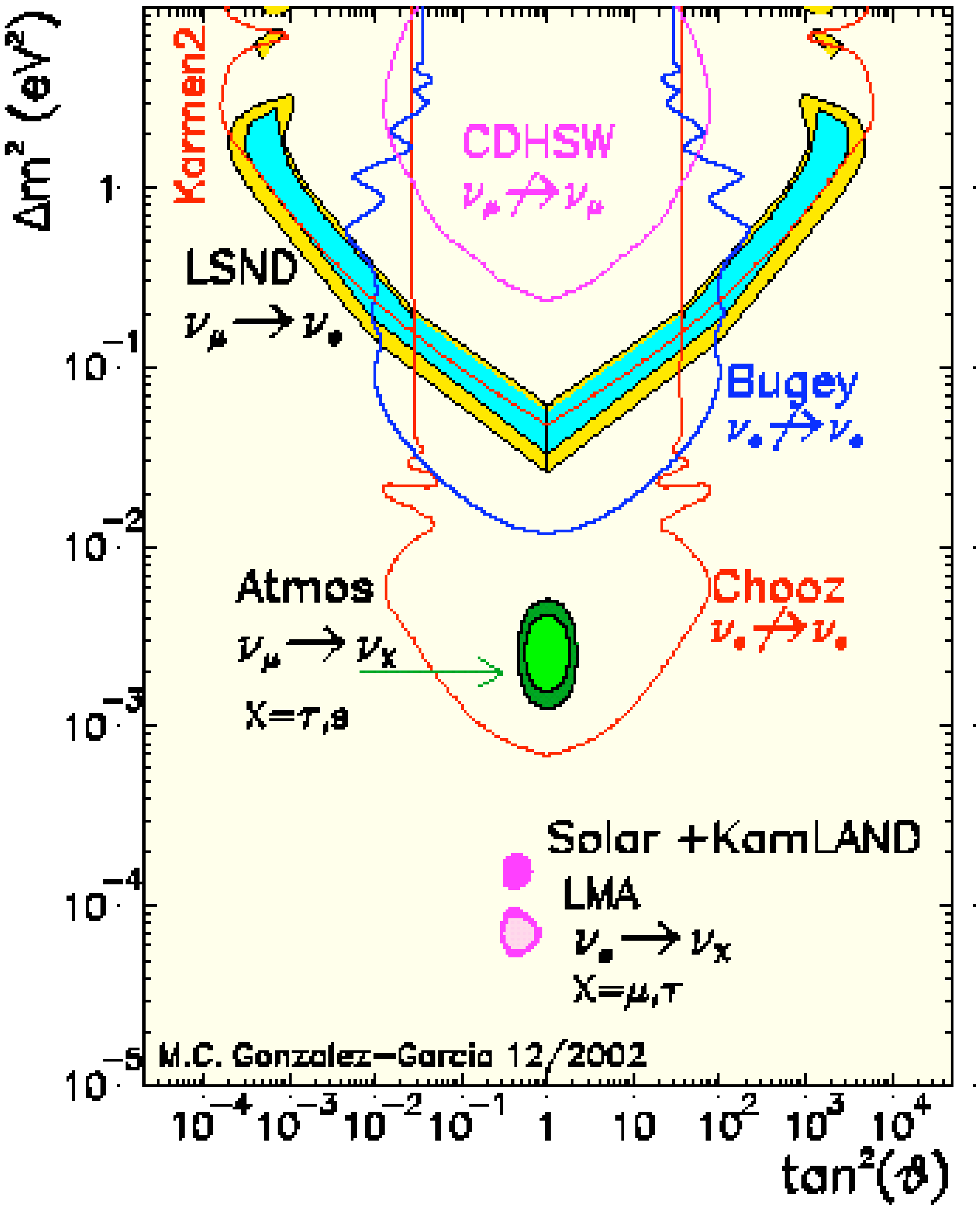
Models of Neutrino Masses and Mixings

G. Altarelli
CERN

Some recent work by our group
G.A., F. Feruglio, I. Masina, hep-ph/0210342
(Addendum: v2 in Nov. '03), hep-ph/0402121.
Reviews:
G.A., F. Feruglio, hep-ph/0206077/0306265

Solid evidence for ν oscillations (+LSND unclear)

$\Delta m^2_{atm} \sim 2.5\ 10^{-3}\ eV^2$,
$\Delta m^2_{sol} \sim 7\ 10^{-5}\ eV^2$
($\Delta m^2_{LSND} \sim 1\ eV^2$)



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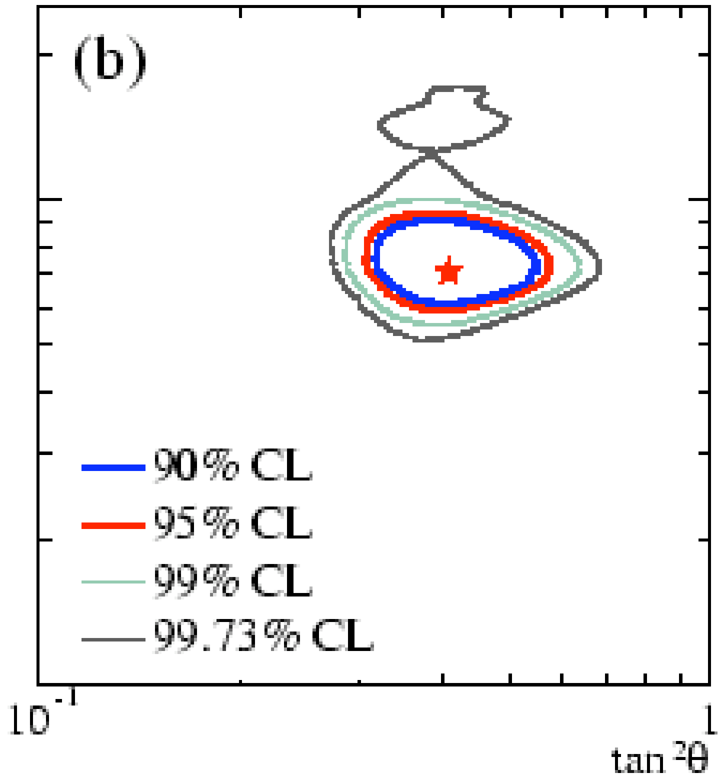
New!

Sept.'03: SNO new results

Salt added to D_2O:
Better NC sensitivity

- Previous results confirmed
- More precision
- The upper Δm^2 part of the LA sol. now disfavoured
- θ_{12} is now 5.4σ from maximal

All data now



ν Oscillations: Summary of Exp. Facts

Homestake, Gallex, Sage, (Super)Kamiokande, Macro... GNO,K2K,..

after KAMLAND, SNO-salt

Atmospheric:

$\Delta m^2_{atm} \sim 2.5 \cdot 10^{-3}$ eV^2
$\sin^2\theta_{23} \sim 1/2$

$\nu_\mu \to \nu_\tau$ dominant
$\nu_\mu \to \nu_e$ small
(Chooz $|U_{13}| <\sim 0.2$)
$\nu_\mu \to \nu_{sterile}$ small

Solar:

The MSW-LA solution selected
$\Delta m^2 \sim 7\ 10^{-5}$ eV^2, $\sin^2\theta_{12} \sim 0.3$

$\nu_e \to \nu_\mu, \nu_\tau$ dominant
$\nu_e \to \nu_{sterile}$ small

LSND:

true or false?
MINIBOONE (in progress) $\nu_\mu \to \nu_e, \nu_{sterile}$

$\Delta m^2 \sim 1$ eV^2, $\sin^2\theta \sim$ small

CPT violation?

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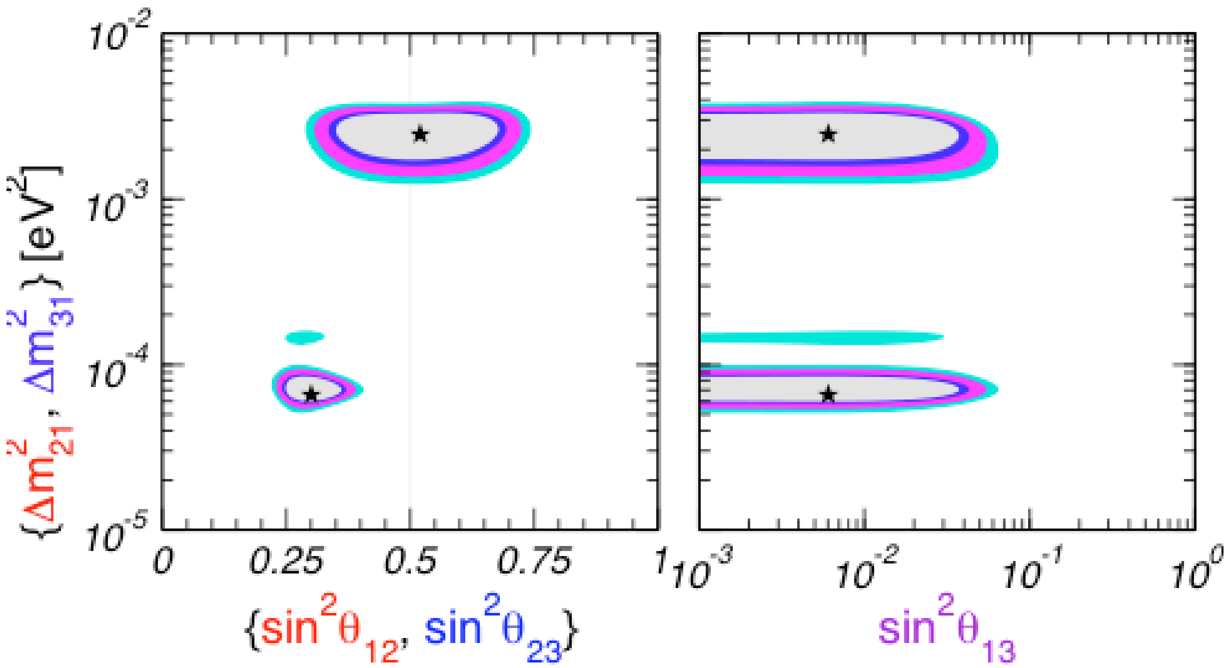
Maltoni et al

parameter	best fit	2σ	3σ	5σ
Δm^2_{21} [10^{-5}eV^2]	6.9	6.0–8.4	5.4–9.5	2.1–28
Δm^2_{31} [10^{-3}eV^2]	2.6	1.8–3.3	1.4–3.7	0.77–4.8
$\sin^2\theta_{12}$	0.30	0.25–0.36	0.23–0.39	0.17–0.48
$\sin^2\theta_{23}$	0.52	0.36–0.67	0.31–0.72	0.22–0.81
$\sin^2\theta_{13}$	0.006	$\lesssim 0.035$	$\lesssim 0.054$	$\lesssim 0.11$



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ν oscillations measure Δm^2. What is $\overline{m}^2$?

$\Delta m^2_{atm} \sim 2.5\ 10^{-3}\ eV^2$; $\Delta m^2_{sun} < \Delta m^2_{atm}$

- Direct limits (PDG '02)

End-point tritium β decay (Mainz)

$$m_{"\nu e"} < 2.8\ eV$$
$$m_{"\nu\mu"} < 170\ KeV$$
$$m_{"\nu\tau"} < 18.2\ MeV$$

- $0\nu\beta\beta$

- Cosmology $\Omega_\nu h^2 \sim \Sigma_i m_i\ /94eV$ $(h^2 \sim 1/2)$

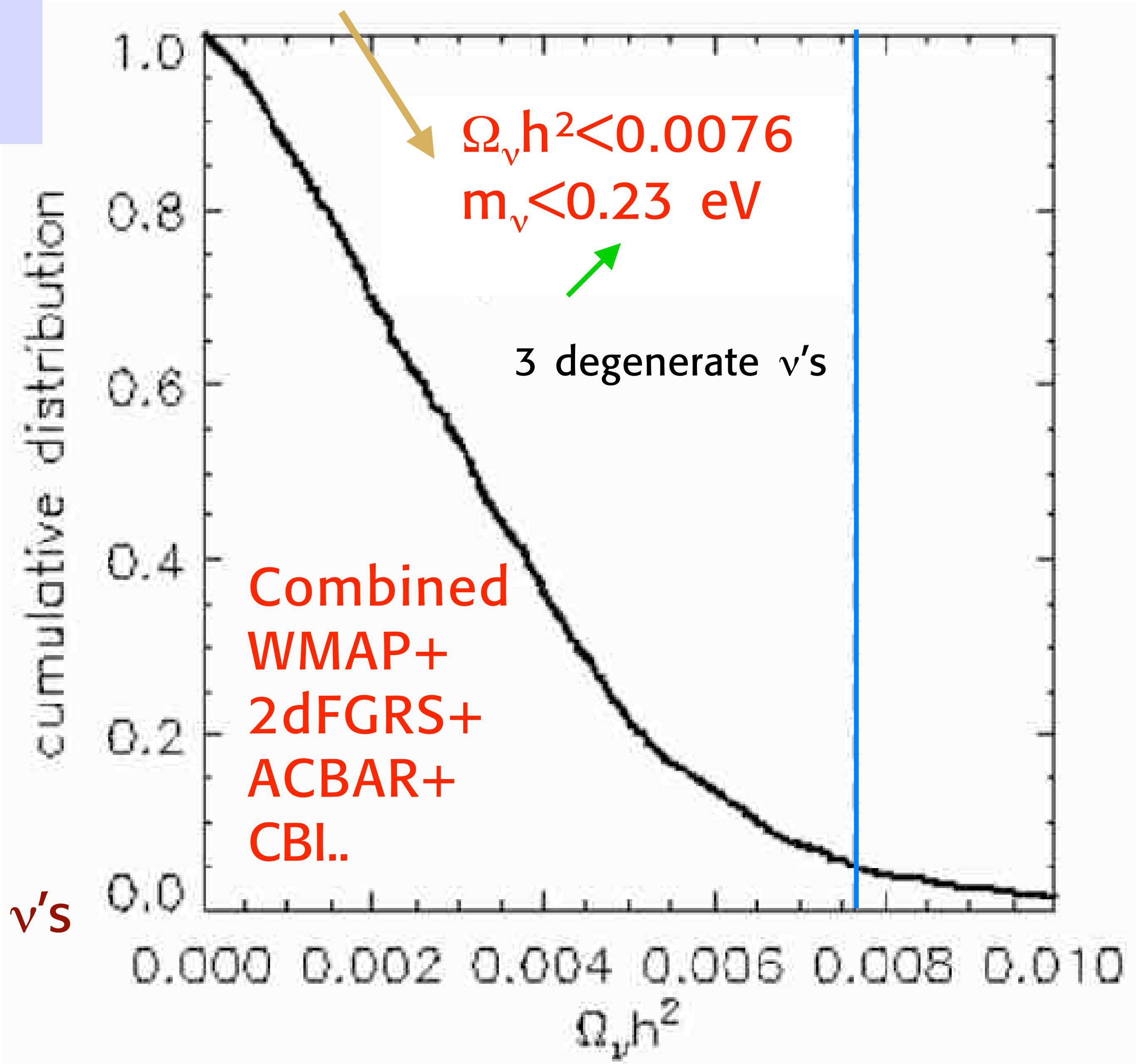
$\Sigma_i m_i$ ~ 0.69 eV (95%) $[\Omega_\nu \sim 0.014]$ WMAP

Any ν mass 0.23-1eV

Why ν's so much lighter than quarks and leptons?

New powerful cosmological limit

Assumes some priors!
Could be somewhat relaxed



All info on the absolute scale of ν mass is very important!

Finding $0\nu\beta\beta$ would also prove Majorana ν's

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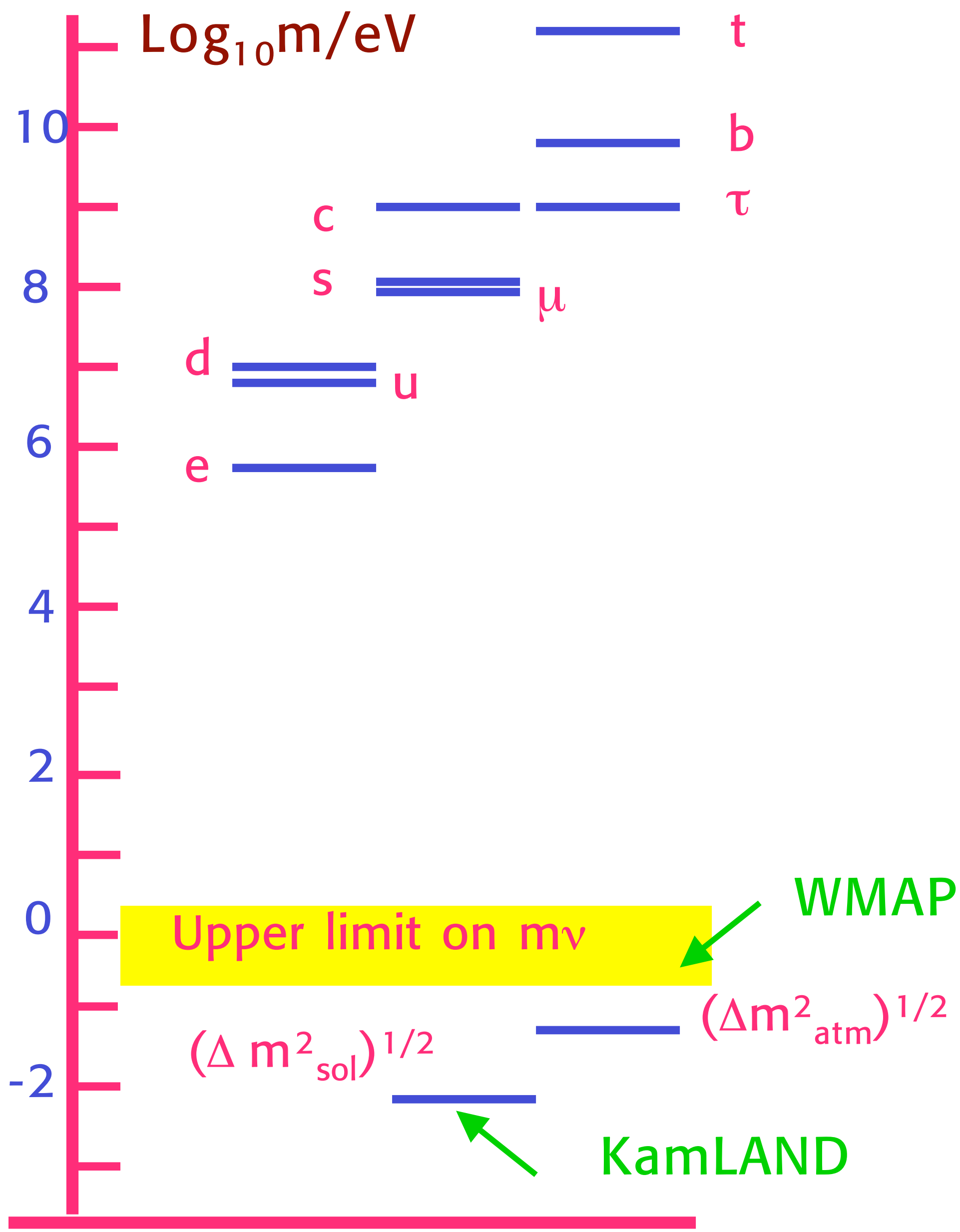

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Neutrino masses are really special!

$m_t/(\Delta m^2_{atm})^{1/2} \sim 10^{12}$

Massless ν's?

- no ν_R
- L conserved

Small ν masses?

- ν_R very heavy
- L not conserved

In general ν mass terms are:

$$\mathcal{L}_\nu = \bar{L} h \nu_R H + \text{h.c.} + \nu_R^T M_R \nu_R + \nu_L^T \frac{\lambda}{M_L} \nu_L HH$$

Dirac
$m_D = h v$
$v = \langle 0|H|0\rangle$

Maiorana

$$m = \frac{\lambda v^2}{M_L}$$

More general see-saw mechanism:

Happy Birthday!!

$$\begin{array}{c|cc} & \nu_L & \nu_R \\ \hline \nu_L & \lambda v^2/M_L & m_D \\ \nu_R & m_D & M_R \end{array}$$

$$m_{light} \sim \frac{m_D^2}{M_R} \quad \text{and/or} \quad \frac{\lambda v^2}{M_L}$$

$m_{heavy} \sim M_R$

$m_{eff} = \nu^T{}_L m_{light} \nu_L$

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A very natural and appealing explanation:

ν's are nearly massless because they are Majorana particles and get masses through L non conserving interactions suppressed by a large scale $M \sim M_{GUT}$

$$m_\nu \sim \frac{m^2}{M}$$

$m \quad m_t \sim v \sim 200$ GeV

M: scale of L non cons.

Note:

$m_\nu \sim (\Delta m^2_{atm})^{1/2} \sim 0.05$ eV

$m \sim v \sim 200$ GeV

$M \sim 10^{15}$ GeV

Neutrino masses are a probe of physics at M_{GUT} !

Baryogenesis   A most attractive possibility:

BG via Leptogenesis near the GUT scale

$T \sim 10^{12\pm3}$ GeV (after inflation)

Buchmuller,Yanagida, Plumacher, Ellis, Lola, Giudice et al, Fujii et al

Only survives if Δ(B-L) is not 0

(otherwise is washed out at T_{ew} by instantons)

Main candidate: decay of lightest ν_R ($M \sim 10^{12}$ GeV)

L non conserv. in ν_R out-of-equilibrium decay:

B-L excess survives at T_{ew} and gives the obs. B asymm.

Quantitative studies confirm that the range of m_i from ν oscill's is compatible with BG via (thermal) LG

In particular the bound was derived

$m_i < 10^{-1}$ eV

Close to WMAP

Buchmuller, Di Bari, Plumacher
Giudice et al

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The current experimental situation is still unclear

- LSND: true or false?
- what is the absolute scale of ν masses?
- …

Different classes of models are possible:

If LSND true

sterile ν(s)??

CPT violat'n??

- "3-1"

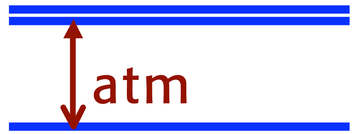
$\nu_{sterile}$

If LSND false → 3 light ν's are OK

We assume this case here

- Degenerate ($m^2 >> \Delta m^2$)

- Inverse hierarchy

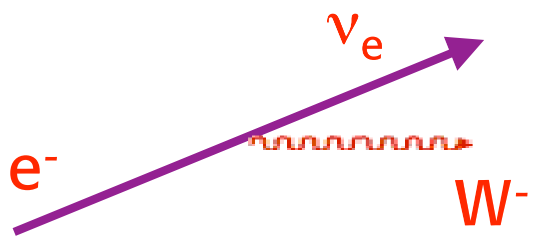


- Normal hierarchy

3-ν Models

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = U \begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

flavour   mass



$U = U_{P\text{-}MNS}$

Pontecorvo
Maki, Nakagawa, Sakata

In basis where e^-, μ^-, τ^- are diagonal:

$$U = \begin{bmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{bmatrix}\begin{bmatrix}c_{13} & 0 & s_{13}e^{-i\delta}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta} & 0 & c_{13}\end{bmatrix}\begin{bmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{bmatrix} \sim$$

s = solar: large

$$\sim \begin{bmatrix}c_{13}\,c_{12} & c_{13}\,s_{12} & s_{13}e^{-i\delta}\\ \ldots & \ldots & c_{13}\,s_{23}\\ \ldots & \ldots & c_{13}\,c_{23}\end{bmatrix}$$

CHOOZ: $|s_{13}| <\sim 0.2$

atm.: ~ max

$$U \approx \begin{bmatrix} c & -s & 0\\ \frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{-1}{\sqrt{2}}\\ \frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}$$

(some signs are conventional)

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$$m_\nu \sim U \begin{bmatrix} e^{i\phi_1} m_1 & 0 & 0 \\ 0 & e^{i\phi_2} m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} U^T$$

In general 9 parameters:
3 masses, 3 angles,
3 phases

$L^T m_\nu L$

For $s_{13} \sim 0$:

$0\nu\beta\beta$

$$m_\nu \sim \begin{bmatrix} m_1 c^2 + m_2 s^2 & (m_1 - m_2) cs/\sqrt{2} & (m_1 - m_2) cs/\sqrt{2} \\ \dots & (m_1 s^2 + m_2 c^2 + m_3)/2 & (m_1 s^2 + m_2 c^2 - m_3)/2 \\ \dots & \dots & (m_1 s^2 + m_2 c^2 + m_3)/2 \end{bmatrix}$$

Note:

- m_ν is symmetric
- phases included in m_i

Relation between masses and frequencies:

$P(\nu_e \text{<->} \nu_\mu) = P(\nu_e \text{<->} \nu_\tau) = 1/2 \sin^2 2\theta_{12} \cdot \sin^2 \Delta_{sun}$

$P(\nu_\mu \text{<->} \nu_\tau) = \sin^2 \Delta_{atm} - 1/4 \sin^2 2\theta_{12} \cdot \sin^2 \Delta_{sun}$

$$\Delta_{sun} = \frac{m_2^2 - m_1^2}{4E} L \quad ; \qquad \Delta_{atm} = \frac{m_3^2 - m_{1,2}^2}{4E} L$$

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In our def.: $\Delta_{sun} > 0$, $\Delta_{atm} >$ or < 0

$0\nu\beta\beta$ can tell degenerate, inverted or normal hierarchy

$$|m_{ee}| = c_{13}^2\,[m_1 c_{12}^2 + e^{i\alpha} m_2 s_{12}^2] + m_3 e^{i\beta} s_{13}^2$$

LA: ~0.3-1

Degenerate: $\sim |m|\,|c_{12}^2 + e^{i\alpha} s_{12}^2|$

$|m_{ee}| \sim |m|\ (0.3\ \text{-}1) < 0.23\text{-}1$ eV

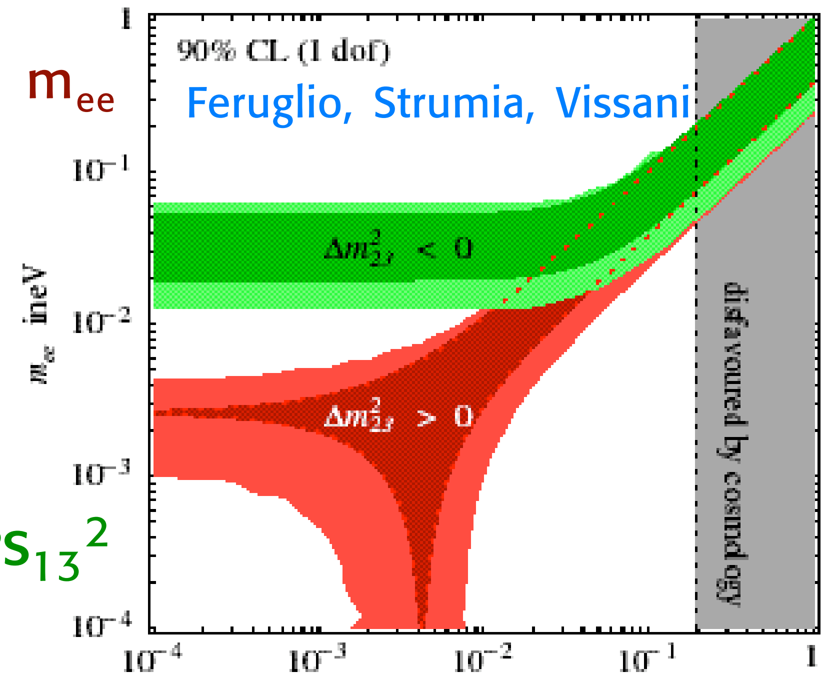
IH: $\sim (\Delta m^2_{atm})^{1/2} |c_{12}^2 + e^{i\alpha} s_{12}^2|$

$|m_{ee}| \sim (1.6\text{-}5)\ 10^{-2}$ eV

NH: $\sim (\Delta m^2_{sol})^{1/2} s_{12}^2 + (\Delta m^2_{atm})^{1/2} e^{i\beta} s_{13}^2$

$|m_{ee}| \sim$ (few) 10^{-3} eV

Full dependence on min m_ν



lightest m_ν (eV)

Present exp. limit: $m_{ee} < 0.3\text{-}0.5$ eV
(and a hint of signal?????)

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Evidence for $0\nu\beta\beta$?

Heidelberg-Moscow
Klapdor-Kleingrothaus et al

Not at all compelling!!!!
1.5σ?, 2.2σ? 3.1σ?

Iff true: (WMAP ??)

$m_{ee}/z = 0.39 \pm 0.11\,\mathrm{eV} >> (\Delta m^2_{atm})^{1/2}$

($z \sim 0.6$-2.8
uncert. matrix element)

would clearly point
to degenerate models

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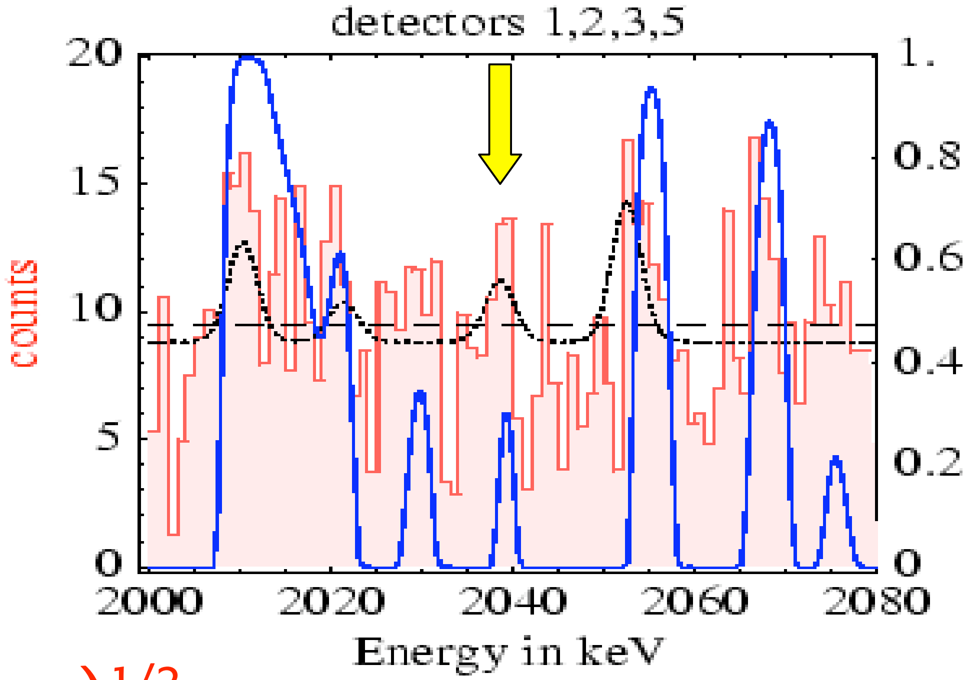
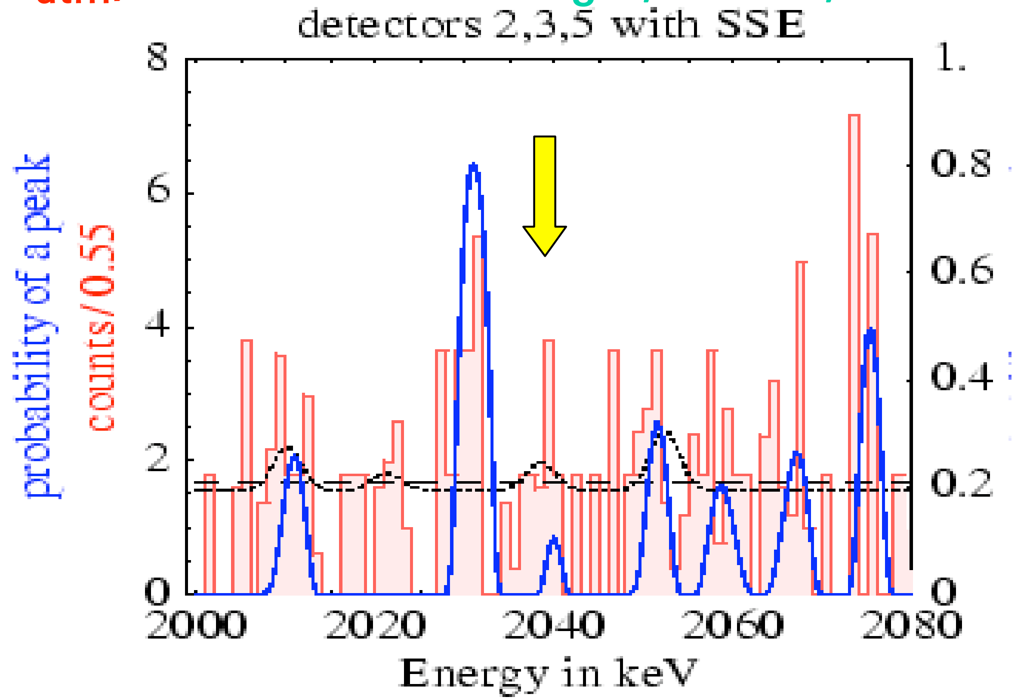


Feruglio, Strumia, Vissani



Degenerate ν's

$m^2 \gg \Delta m^2$

- Apriori compatible with hot dark matter ($m \sim 1$-2 eV) → was considered by many
- Limits on m_{ee} from $0\nu\beta\beta$ then imply large mixing also for solar oscillations: (Vissani; Georgi,Glashow)

$m_{ee} < 0.3$-0.5 eV (Exp)

$$m_{ee} = c^2_{13}\,(m_1 c^2_{12} + m_2 s^2_{12}) + s^2_{13} m_3 \sim m_1 c^2_{12} + m_2 s^2_{12}$$

If $|m_1| \sim |m_2| \sim |m_2| \sim 1$-$2$ eV ⟶ $m_1 = -m_2$ and $c^2_{12} \sim s^2_{12}$

LA solution: $\sin^2\theta \sim 0.3$ ⟶ $\cos^2\theta - \sin^2\theta \sim 0.4$

a moderate suppression factor!

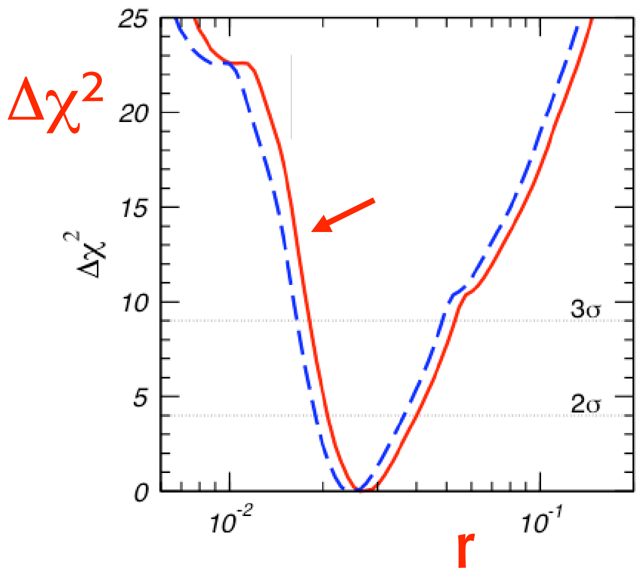
Trusting WMAP: $|m| < 0.23$ eV, only a moderate degeneracy is allowed: for LA, $m/(\Delta m^2_{atm})^{1/2} < 5$, $m/(\Delta m^2_{sol})^{1/2} < 30$.

Less constraints from $0\nu\beta\beta$ (both $m_1 = \pm m_2$ allowed)

Recall: leptogenesis prefers $|m| < 0.1$ eV

After KamLAND, SNO and WMAP not too much hierarchy is needed for ν masses:

$r \sim \Delta m^2_{sol}/\Delta m^2_{atm} \sim 1/40$



Precisely at 3σ: $0.018 < r < 0.053$

or

$m_{heaviest} < 1 - 0.23$ eV

$m_{next} > \sim 7\ 10^{-3}$ eV

Anarchical or semi-anarchical models

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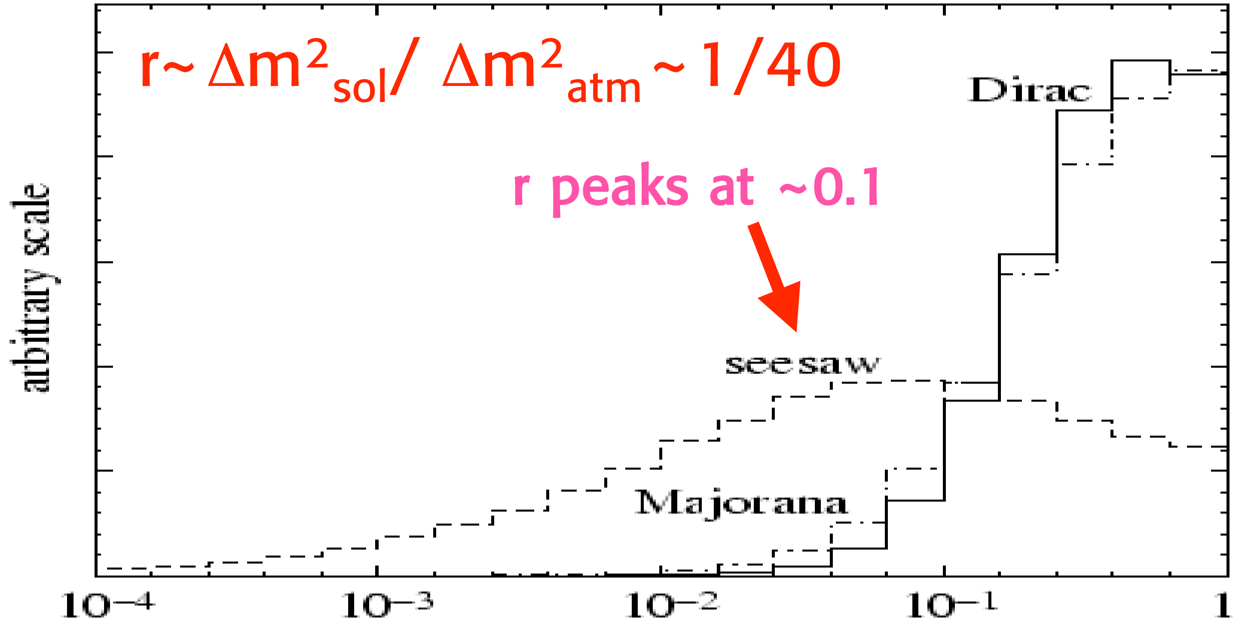
Anarchy (or accidental hierarchy):
No structure in the leptonic sector

Hall, Murayama, Weiner

See-Saw:
$m_\nu \sim m^2/M$
produces hierarchy from random m,M

could fit LA



But: all mixing angles should be large

marginal for LA →
predicts θ_{13} near bound

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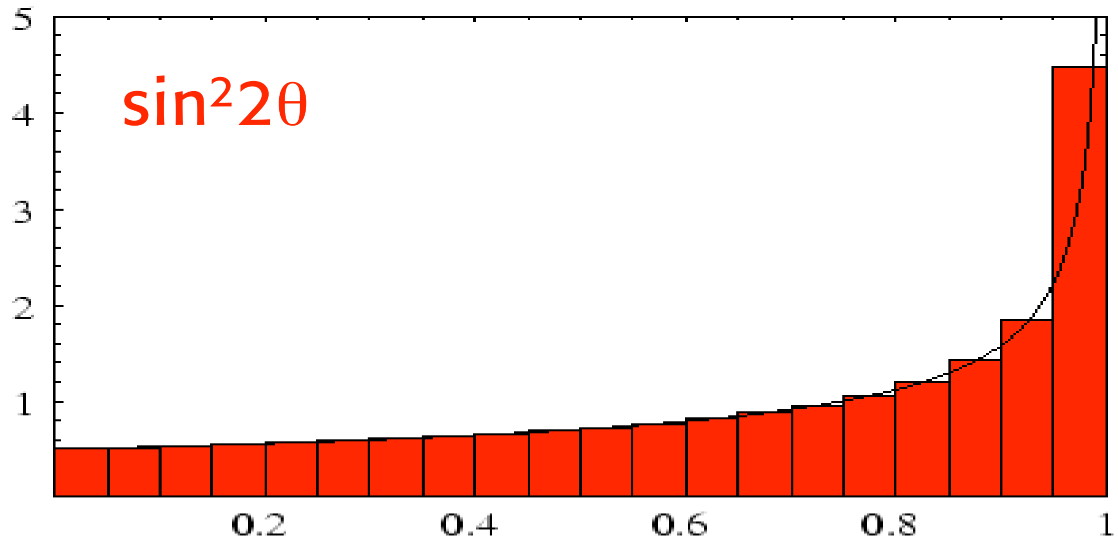


Semianarchy: no structure in 23

Consider a matrix like $m_\nu \sim \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}$

Note: $\theta_{13} \sim \lambda$
$\theta_{23} \sim 1$

with coeff.s of o(1) and det23~o(1)
[λ~1 corresponds to anarchy]

After 23 and 13 rotations $m_\nu \sim \begin{pmatrix} \lambda^2 & \lambda & 0 \\ \lambda & \eta & 0 \\ 0 & 0 & 1 \end{pmatrix}$

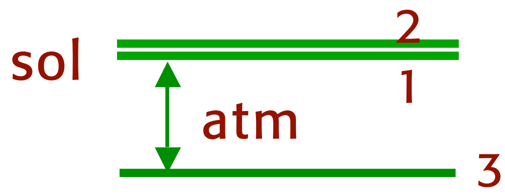
Normally two masses are of o(1) and $\theta_{12} \sim \lambda$
But if, accidentally, $\eta \sim \lambda$, then the solar angle is also large.

The advantage over anarchy is that θ_{13} is small, but the hierarchy $m^2_3 >> m^2_2$ is accidental

G. Altarelli Ramond et al, Buchmuller et al

Inverted Hierarchy

Zee, Joshipura et al;
Mohapatra et al; Jarlskog et al;
Frampton,Glashow; Barbieri et al
Xing; Giunti, Tanimoto



$m^2 \sim 10^{-3}\ eV^2$

An interesting model for double maximal mixing (bimixing)

$$U \sim \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/2 & 1/2 & 1/\sqrt{2} \end{bmatrix}$$

1st approximation

$$m_{\nu diag} = \begin{bmatrix} m & 0 & 0 \\ 0 & -m & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad ; \quad U m_{\nu diag} U^T = 1/\sqrt{2} \begin{bmatrix} 0 & m & m \\ m & 0 & 0 \\ m & 0 & 0 \end{bmatrix}$$

Can arise from see-saw or dim-5 $L^T H H^T L$

e.g. by approximate L_e-L_μ-L_τ symmetry

- 1-2 degeneracy stable under rad. corr.'s

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1^{st} approximation

$$m_{\nu diag} = \begin{bmatrix} m & 0 & 0 \\ 0 & -m & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad ; \quad U m_{\nu diag} U^T = 1/\sqrt{2} \begin{bmatrix} 0 & m & m \\ m & 0 & 0 \\ m & 0 & 0 \end{bmatrix}$$

- LA? This texture prefers θ_{sol} closer to maximal than θ_{atm} i.e $\theta_{sol} - \pi/4$ small for $(\Delta m^2_{sol}/\Delta m^2_{atm})_{LA} \sim 1/40$

In fact: 12-> $\begin{bmatrix} 0 & m \\ m & 0 \end{bmatrix} \rightarrow$ Pseudodirac θ_{12} maximal 23-> $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \rightarrow \theta_{23} \sim o(1)$

With perturbations: $\begin{bmatrix} 0 & m & m \\ m & 0 & 0 \\ m & 0 & 0 \end{bmatrix} \longrightarrow m \begin{bmatrix} \delta & 1 & 1 \\ 1 & \eta & \eta \\ 1 & \eta & \eta \end{bmatrix}$

$tg^2\,\theta_{12} \sim 1 + o(\delta + \eta) \qquad (\Delta m^2_{sol}/\Delta m^2_{atm})_{LA} \sim o(\delta + \eta)$

- In principle one can use the charged lepton mixing to go away from θ_{12} maximal. In practice constraints from θ_{13} small $(\delta\theta_{12} \sim \theta_{13})$

Frampton et al; GA, Feruglio, Masina '04

For the corrections to bimixing from the charged lepton sector, typically $|\sin\theta_{13}| \sim (1 - \tan^2\theta_{12})/4$

GA, Feruglio, Masina '04

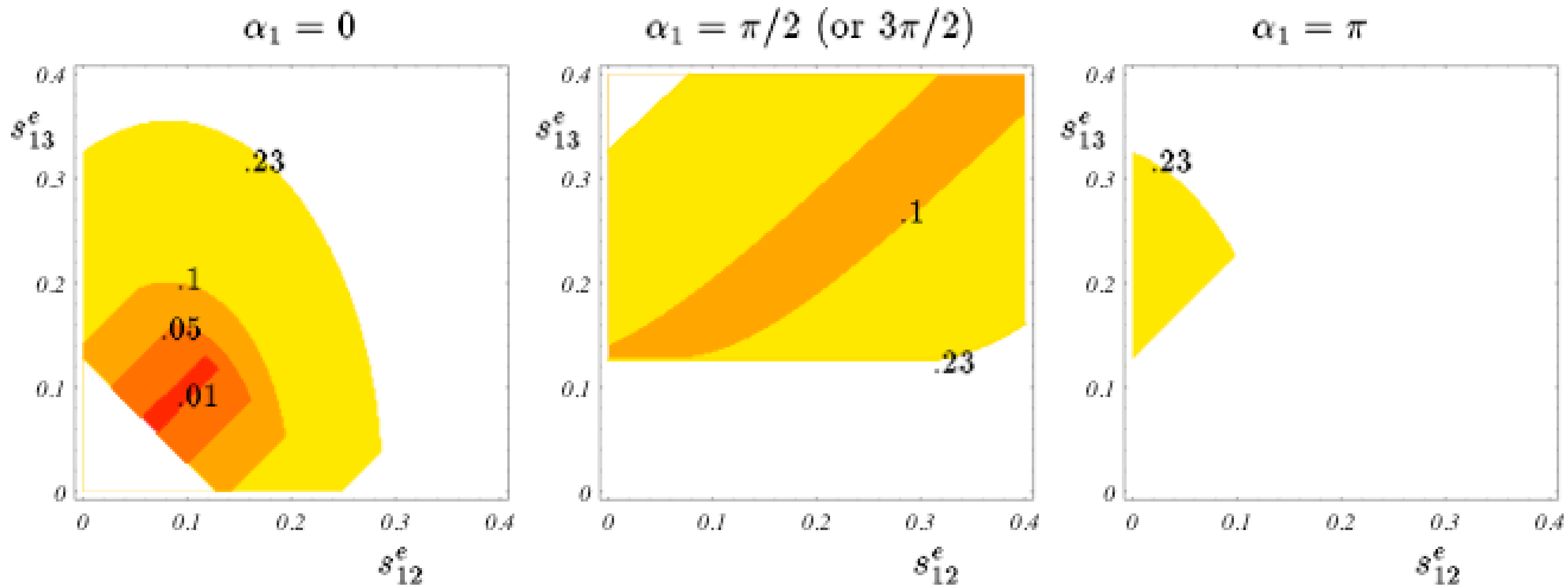

Figure 1: Taking an upper bound on $|U_{e3}|$ respectively equal to $0.23, 0.1, 0.05, 0.01$, we show (from yellow to red) the allowed regions of the plane $[s^e_{12}, s^e_{13}]$. Each plot is obtained by setting α_1 to a particular value, while leaving $\alpha_2 + \delta_e$ free. We keep the present 3 σ window for δ_{sol} [10].

- In general more θ_{12} is close to maximal, more is IH likely

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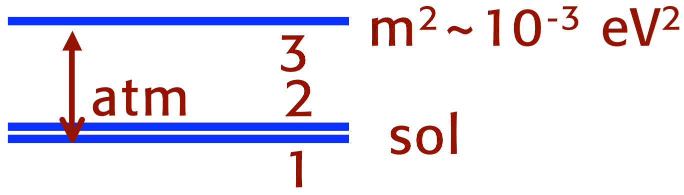
Normal Hierarchy



- Assume 3 widely split light neutrinos.
- For u, d and l^- Dirac matrices the 3^{rd} generation eigenvalue is dominant.
- May be this is also true for $m_{\nu D}$: diag $m_{\nu D} \sim (0,0,m_{D3})$.
- Assume see-saw is dominant: $m_\nu \sim m^T_D M^{-1} m_D$
  See-saw quadratic in m_D: tends to enhance hierarchy
- Maximally constraining: GUT's relate q, l^-, ν masses!

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- A crucial point: in the 2-3 sector we need both large m_3-m_2 splitting and large mixing.

  $m_3 \sim (\Delta m^2_{atm})^{1/2} \sim 5\ 10^{-2}$ eV
  $m_2 \sim (\Delta m^2_{sol})^{1/2} \sim 7\ 10^{-3}$ eV for LA

- The "theorem" that large Δm_{32} implies small mixing (pert. th.: $\theta_{ij} \sim 1/|E_i-E_j|$) is not true in general: all we need is (sub)det[23]~0

- Example: $m_{23} \sim \begin{bmatrix} x^2 & x \\ x & 1 \end{bmatrix}$

  Det = 0; Eigenvl's: 0, $1+x^2$
  Mixing: $\sin^2 2\theta = 4x^2/(1+x^2)^2$

So all we need are natural mechanisms for det[23]=0

For x~1
large splitting
and large mixing!

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Examples of mechanisms for Det[23]~0

see-saw $m_\nu \sim m^T_D M^{-1} m_D$

1) A ν_R is lightest and coupled to μ and τ

King; Allanach; Barbieri et al......

$$M \sim \begin{bmatrix} \varepsilon & 0 \\ 0 & 1 \end{bmatrix} \longrightarrow M^{-1} \sim \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 1 \end{bmatrix} \approx \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 0 \end{bmatrix}$$

$$m_\nu \sim \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1/\varepsilon & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix} \approx 1/\varepsilon \begin{bmatrix} a^2 & ac \\ ac & c^2 \end{bmatrix}$$

2) M generic but m_D "lopsided" $m_D \sim \begin{bmatrix} 0 & 0 \\ x & 1 \end{bmatrix}$

Albright, Barr; GA, Feruglio, ...

$$m_\nu \sim \begin{bmatrix} 0 & x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ b & c \end{bmatrix} \begin{bmatrix} 0 & 0 \\ x & 1 \end{bmatrix} = c \begin{bmatrix} x^2 & x \\ x & 1 \end{bmatrix}$$

Caution: if 0 -> 0(ε), det23=0 could be spoiled by suitable $1/\varepsilon$ terms in M^{-1}

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An important property of SU(5)

Left-handed quarks have small mixings (V_{CKM}),
but right-handed quarks can have large mixings (unknown).

In SU(5): LH for d quarks $\Longleftrightarrow$ RH for l^- leptons

$$m_d \sim \overbrace{\bar{d}_R}^{\bar{5}}\, \overbrace{d_L}^{10}$$

$$m_e \sim \overbrace{\bar{e}_R}^{10}\, \overbrace{e_L}^{\bar{5}}$$

$$\bar{5} : (\underbrace{\bar{d},\bar{d},\bar{d}}_{R},\ \underbrace{\nu,e^-}_{L})$$

$$\boxed{m_d = m_e^T}$$

cannot be exact, but approx.

Most "lopsided" models are based on this fact. In these models large atmospheric mixing arises (at least in part) from the charged lepton sector.

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- Hierarchical ν's and see-saw dominance

$$L^T m_\nu L \rightarrow m_\nu \sim m_D^2/M$$

allow to relate q, l, ν masses and mixings in GUT models.
For dominance of dim-5 operators -> less constraints

$$\lambda^2/M\, L^T L H H \rightarrow m_\nu \sim \lambda^2 v^2/M$$

- The correct pattern of masses and mixings, also including ν's, is obtained in simple models based on

$SU(5)xU(1)_{flavour}$

Ramond et al; GA, Feruglio+Masina; Buchmuller et al; King et al; Yanagida et al, Berezhiani et al; Lola et al.......

- $SO(10)$ models could be more predictive, as are non abelian flavour symmetries, eg O(3)

Albright, Barr; Babu et al; Buccella et al; Barbieri et al; Raby et al; King, Ross

- The non trivial pattern of fermion masses and mixing demands a flavour structure (symmetry)

- (SUSY) $SU(5)XU(1)_F$ models offer a minimal description of flavour symmetry

- A flexible enough framework used to realize and compare models with anarchy or hierarchy (direct or inverse) in ν sector, with see-saw dominance or not.

- On this basis we found that for LA there is still a significant preference for hierarchy vs anarchy

G.A., F. Feruglio, I. Masina, hep-ph/0210342 (v2 Nov '03)

Previous related work: Haba,Murayama; Hirsch,King; Vissani; Rosenfeld,Rosner; Antonelli et al.…

Hierarchy for masses and mixings via horizontal U(1) charges.

Froggatt, Nielsen '79

Principle: A generic mass term

$$\bar{R}_1 m_{12} L_2 H$$

is forbidden by U(1)
if $q_1+q_2+q_H = 0$

q_1, q_2, q_H:
U(1) charges of
$\bar{R}_1, L_2, H$

U(1) broken by vev of "flavon" field θ with U(1) charge q_θ= -1.
The coupling is allowed: if vev $\theta = w$, and $w/M=\lambda$ we get:

Δ_{charge}

$$\bar{R}_1 m_{12} L_2 H\ (\theta/M)^{q1+q2+qH} \qquad m_{12} \to m_{12}\ \lambda^{q1+q2+qH}$$

Hierarchy: More Δ_{charge} -> more suppression (λ small)

One can have more flavons (λ, λ', ...)
with different charges (>0 or <0)etc -> many versions

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With suitable charge assignments all relevant patterns can be obtained

1st fam. 2nd 3rd

Ψ_{10}: (5, 3, 0)
Ψ_5: (2, 0, 0)
Ψ_1: (1,-1, 0)

Equal 2,3 ch. for lopsided

Recall: $u \sim 10\ 10$
$d = e^T \sim \bar{5}\ 10$
$\nu_D \sim \bar{5}\ 1$; $M_{RR} \sim 1\ 1$

No structure for leptons

No automatic det23 = 0

Automatic det23 = 0

Model	Ψ_{10}	$\Psi_{\bar{5}}$	Ψ_1	(H_u, H_d)
Anarchical (A)	(3,2,0)	(0,0,0)	(0,0,0)	(0,0)
Semi-Anarchical (SA)	(2,1,0)	(1,0,0)	(2,1,0)	(0,0)
Hierarchical (H_I)	(6,4,0)	(2,0,0)	(1,-1,0)	(0,0)
Hierarchical (H_{II})	(5,3,0)	(2,0,0)	(1,-1,0)	(0,0)
Inversely Hierarchical (IH_I)	(3,2,0)	(1,-1,-1)	(-1,+1,0)	(0,+1)
Inversely Hierarchical (IH_{II})	(6,4,0)	(1,-1,-1)	(-1,+1,0)	(0,+1)

all charges positive

not all charges positive

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Example: Normal Hierarchy

G.A., Feruglio, Masina

1st fam. 2nd 3rd

q(10): (5, 3, 0)
q($\bar{5}$): (2, 0, 0)
q(1): (1,-1, 0)

Note: not all charges positive
--> det23 suppression

$q(H) = 0,\ q(\bar{H}) = 0$
$q(\theta) = -1,\ q(\theta') = +1$

In first approx., with $<\theta>/M \sim \lambda \sim \lambda' \sim 0.35 \sim o(\lambda_C)$

$10_i 10_j$

$$m_u \sim v_u \begin{bmatrix} \lambda^{10} & \lambda^8 & \lambda^5 \\ \lambda^8 & \lambda^6 & \lambda^3 \\ \lambda^5 & \lambda^3 & 1 \end{bmatrix},$$

$10_i \bar{5}_j$

$$m_d = m_e^T \sim v_d \begin{bmatrix} \lambda^7 & \lambda^5 & \lambda^5 \\ \lambda^5 & \lambda^3 & \lambda^3 \\ \lambda^2 & 1 & 1 \end{bmatrix}$$

"lopsided"

$\bar{5}_i 1_j$

$$m_{\nu D} \sim v_u \begin{bmatrix} \lambda^3 & \lambda & \lambda^2 \\ \lambda & \lambda' & 1 \\ \lambda & \lambda' & 1 \end{bmatrix},$$

$1_i 1_j$

$$M_{RR} \sim M \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda'^2 & \lambda' \\ \lambda & \lambda' & 1 \end{bmatrix}$$

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Note: coeffs. 0(1) omitted, only orders of magnitude predicted

All entries are a given power of λ times a free o(1) coefficient

$$m_u \sim v_u \begin{bmatrix} \lambda^{10} & \lambda^8 & \lambda^5 \\ \lambda^8 & \lambda^6 & \lambda^3 \\ \lambda^5 & \lambda^3 & 1 \end{bmatrix}$$

In a statistical approach we generate these coeff.s as random complex numbers $\rho e^{i\phi}$ with ϕ =[0,2π] and ρ= [0.5,2] (default) or [0.8,1.2], or [0.95,1.05] or [0,1]
(real numbers also considered for comparison)

For each model we evaluate the success rate (over many trials) for falling in the exp. allowed window:

(boundaries ~3σ limits)

Maltoni et al, hep-ph/0309130

$r \sim \Delta m^2_{sol}/\Delta m^2_{atm}$

$0.018 < r < 0.053$
$|U_{e3}| < 0.23$
$0.30 < \tan^2\theta_{12} < 0.64$
$0.45 < \tan^2\theta_{23} < 2.57$

for each model the λ,λ' values are optimised

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The optimised values of λ are of the order of λ_C or a bit larger (moderate hierarchy)

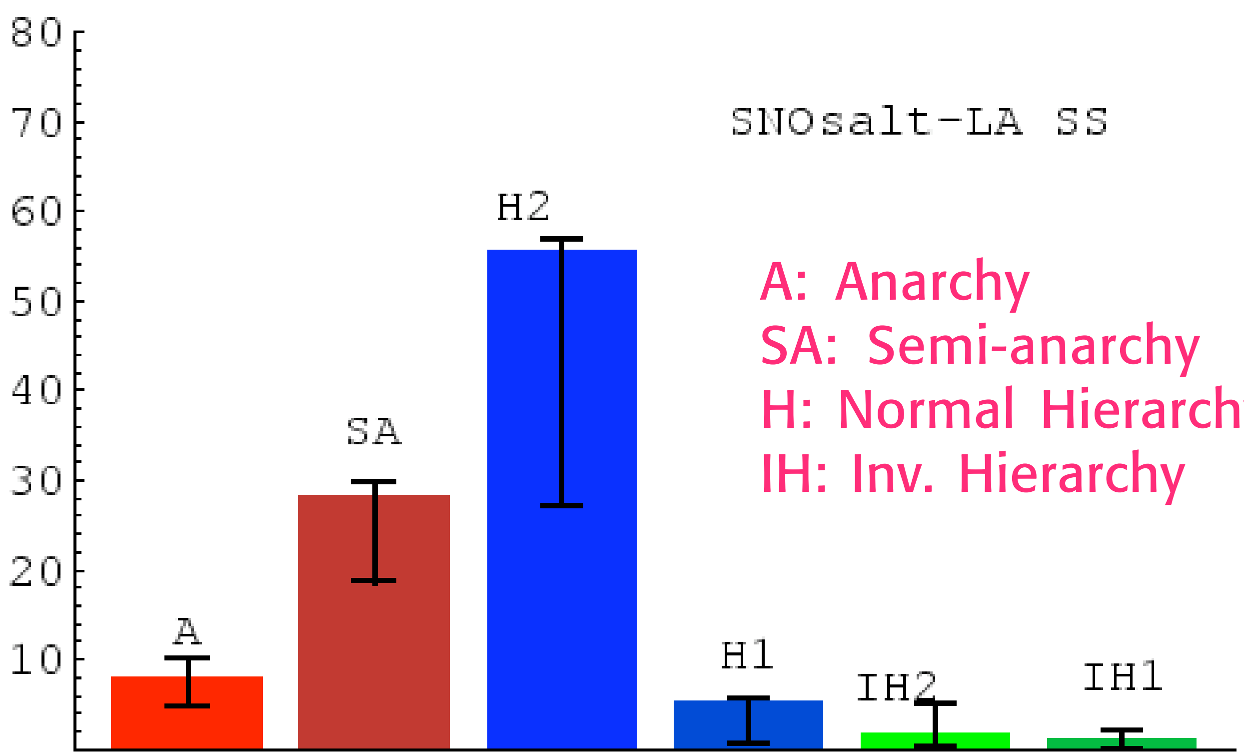
`model`	$\lambda(=\lambda')$
A_{SS}	0.2
SA_{SS}	0.25
$H_{(SS,II)}$	0.35
$H_{(SS,I)}$	0.45
$IH_{(SS,II)}$	0.45
$IH_{(SS,I)}$	0.25

Results with see-saw dominance (updated in Nov. '03):

Scale: Σrates=100

1 or 2 refer to models with 1 or 2 flavons of opposite ch.

With charges of both signs and 1 flavon some entries are zero



A: Anarchy
SA: Semi-anarchy
H: Normal Hierarchy
IH: Inv. Hierarchy

Errors are linear comb. of stat. and syst. errors (varying the extraction procedure: interval of ρ, real or complex)

H2 is better than SA, better than A, better than IH

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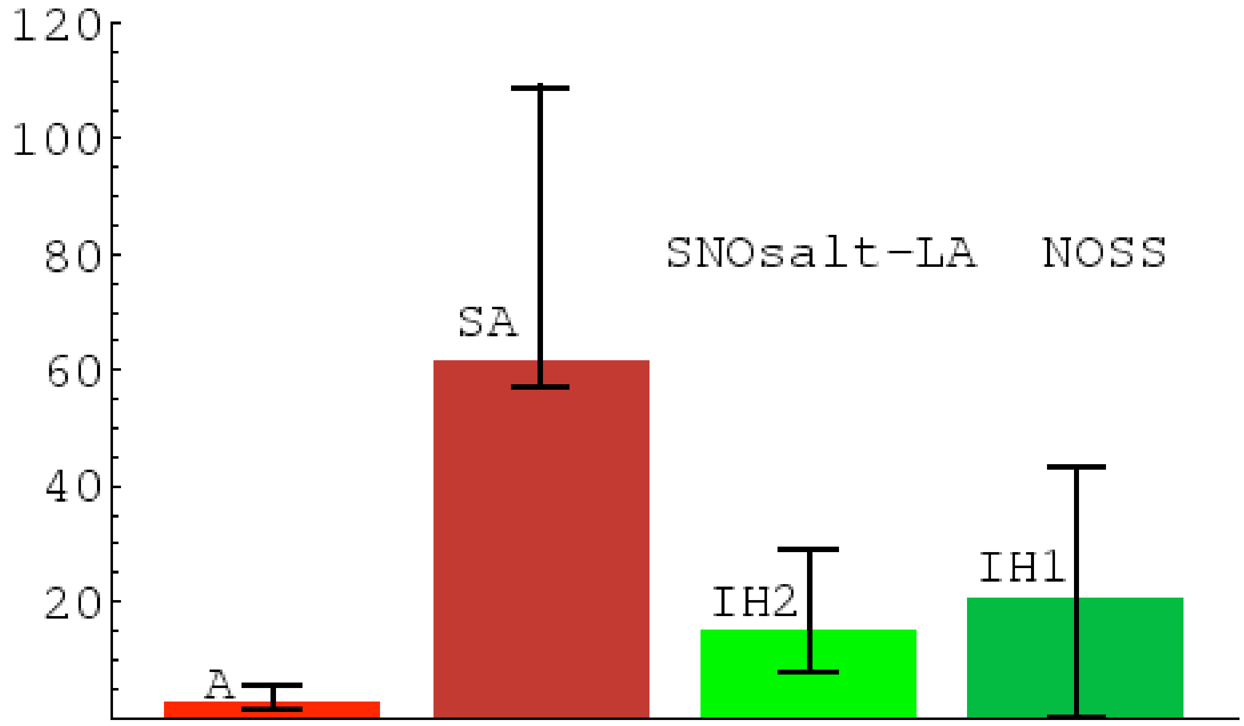
With no see-saw (m_ν generated directly from $L^T m_\nu L \sim \bar{5}\,\bar{5}$)
IH is better than A

[With no-see-saw H coincide with SA]



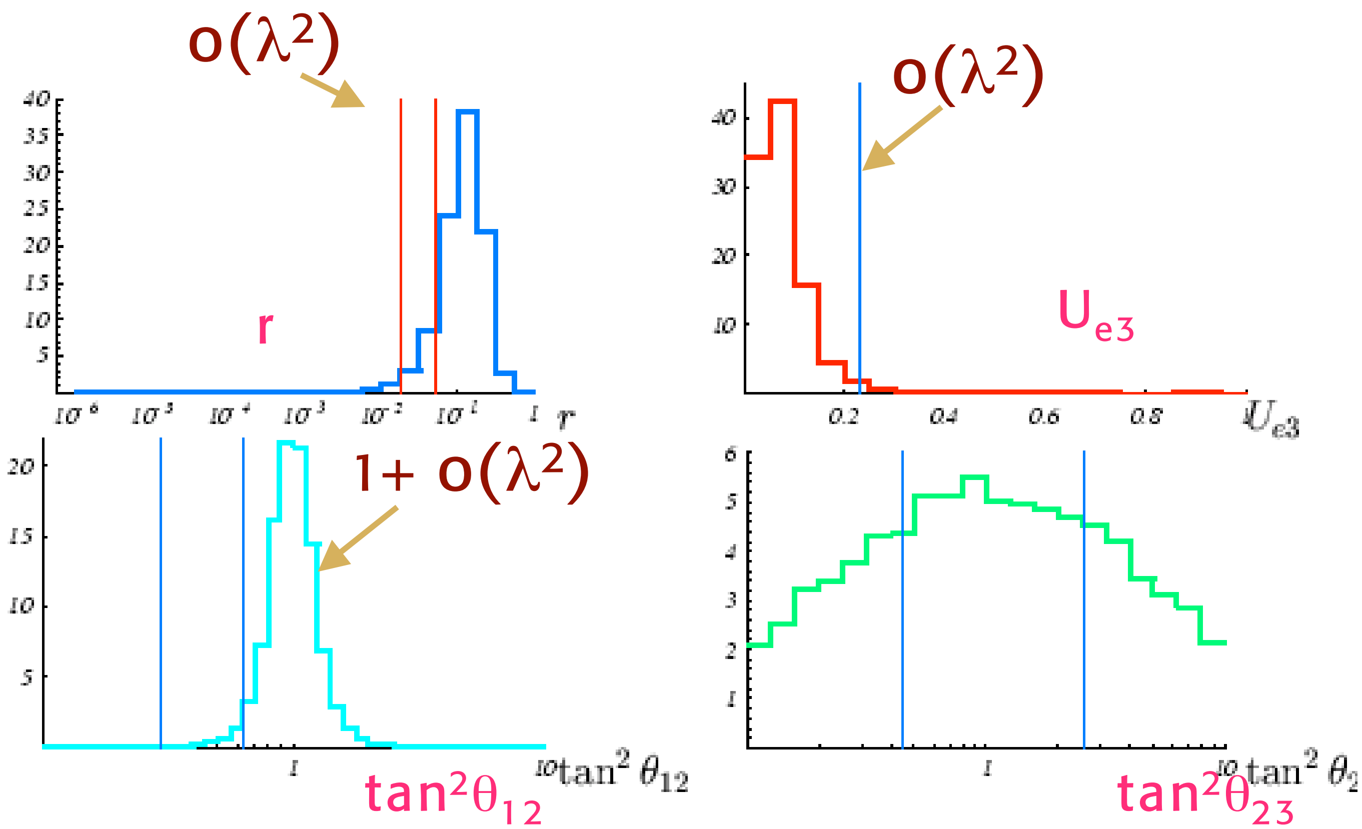
Note: we always include the effect of diagonalising charged leptons

Some distributions

IH2 NO-SS $\lambda=\lambda'=0.3$

We see that IH tends to predict maximal solar mixing angle θ_{12}

Only compatible because of ch. lepton diagonalisation



With data dritfing away from maximal θ_{12}, IH is rapidly disfavoured (in U(1) models)

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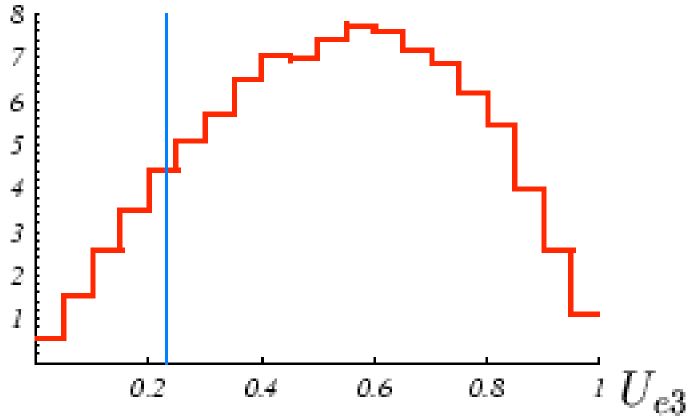
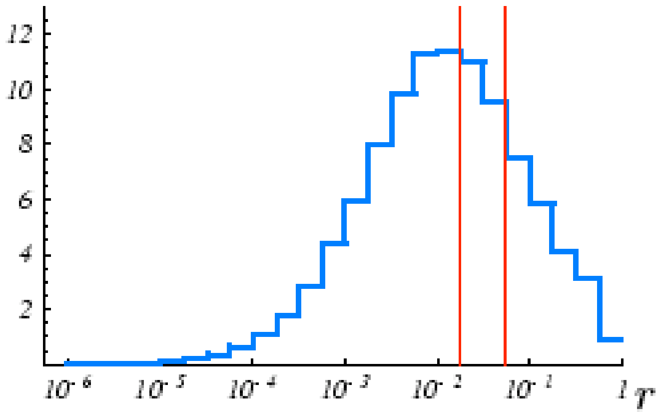
ch. lepton mixing small because m_e small

The main problem
of Anarchy is U_{e3} (as expected)

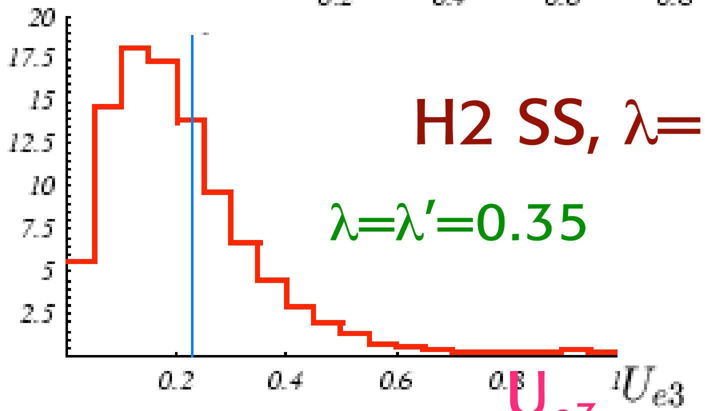
$\lambda=\lambda'=0.2$



In all models the distr.
for $\tan^2\theta_{23}$ is flat



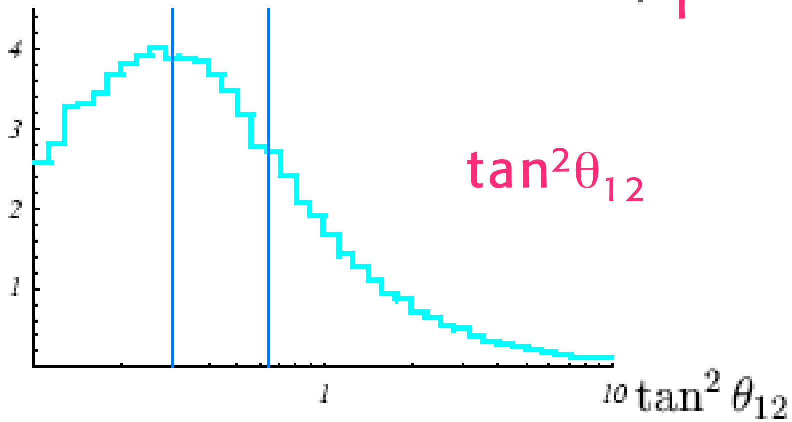
r



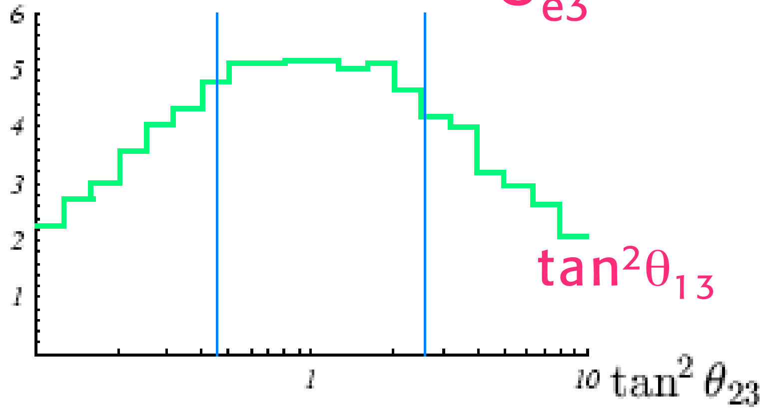
H2 SS, $\lambda=0.35$

$\lambda=\lambda'=0.35$

U_{e3}



$\tan^2\theta_{12}$



$\tan^2\theta_{13}$

G.

The main advantage of SA vs A is for U_{e3}

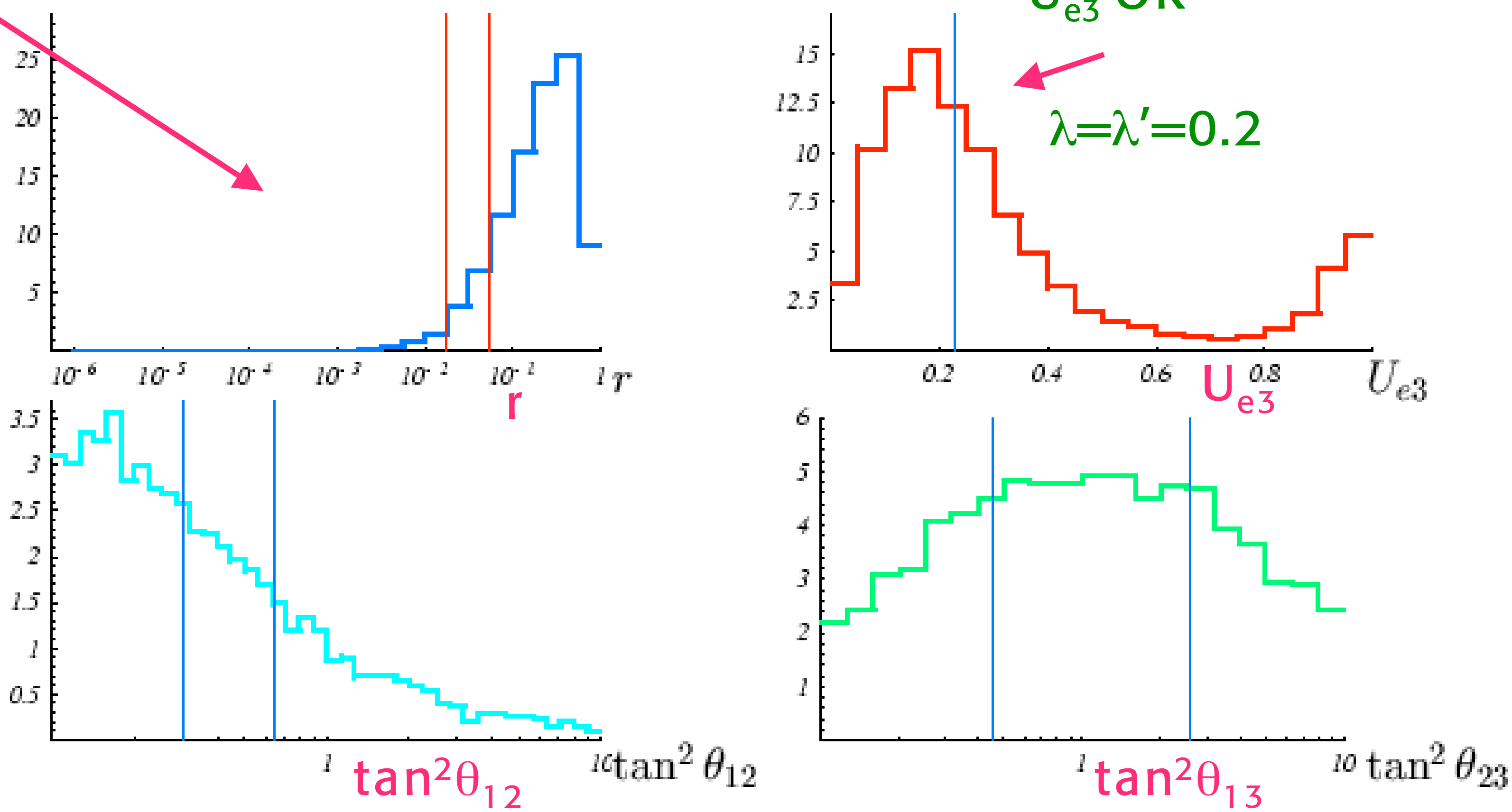
$$m_\nu \sim \begin{bmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{bmatrix}$$

$$r \equiv \frac{\Delta m^2_{sun}}{\Delta m^2_{atm}}\,.$$

$\Psi_5 \sim (2,0,0) \longrightarrow$ $Det_{23} \sim 0(1)$

$SA_{(NOSS)},\ \lambda = 0.2$

works
when
r is small
enough by
chance



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Can ν mixings arise only from the charged lepton sector?

G.A., Feruglio, Masina '04

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = U \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix} \Longrightarrow U = U_e^+ U_\nu$$

flavour mass

diag of ch leptons

$m_\nu = U^* \, m_\nu^{diag} U^+$

$m_e = V_e m_e^{diag} U_e^+$ $\begin{cases} \bar{R} m_e L \\ L_{diag} = U_e L \\ R_{diag} = V_e R \end{cases}$

Assume that, in the lagrangian basis where all symmetries are specified, we have: $U_\nu \sim 1$. Then: $U \sim U_e^+ \sim$

$$\begin{bmatrix} c & -s & 0 \\ \frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

(small effects like s_{13} can be thought to arise from U_ν- 1. Phases dropped for simplicity)

Given $m_e^{diag} \sim m_\tau diag[0,\eta,1]$ (with $\eta = m_\mu/m_\tau$) we obtain:

$$m_e = V_e m_e^{diag} U \sim V_e m_\tau \begin{bmatrix} 0 & 0 & 0 \\ \frac{s\eta}{\sqrt{2}} & -\frac{c\eta}{\sqrt{2}} & \frac{\eta}{\sqrt{2}} \\ -\frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

For $V_e \sim 1$ this is a generalisation of lopsided (s large) but with $det_{12}=0$

Independent of V_e:

$$m_e^+ m_e \sim U^+ (m_e^{diag})^2 U \sim m_\tau^2 \frac{1+\eta^2}{2} \cdot \begin{bmatrix} s^2 & -cs & -s(1-2\eta^2) \\ -cs & c^2 & c(1-2\eta^2) \\ -s(1-2\eta^2) & c(1-2\eta^2) & 1 \end{bmatrix}$$

- all matrix elements of same order (because s is large) "democratic" (hierarchy of masses non trivial)
- $s_{13}=0$ (i.e. eigenvector $(c,s,0)^T$) -> first two columns proportional

Note: in minimal SU(5) models $m_e = m_d^T$. This implies $V_e = U_d$

Quark mixings are small: $V_{CKM} = U_u^+ U_d$

Two possibilities:

- Both U_u and U_d nearly diagonal -> $V_e \sim 1$
- $U_u \sim U_d$ nearly equal and non diagonal

This is the way of democratic models:

$U_u \sim U_d \sim U_e$ -> $V_e \sim U_e$

$V_e \sim 1$

$$m_e = m_\tau \begin{bmatrix} 0 & 0 & 0 \\ \frac{s\eta}{\sqrt{2}} & -\frac{c\eta}{\sqrt{2}} & \frac{\eta}{\sqrt{2}} \\ -\frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$V_e \sim U_e$

$$m_e = m_\tau \frac{1+\eta}{2} \cdot \begin{bmatrix} s^2 & -cs & -s(1-2\eta) \\ -cs & c^2 & c(1-2\eta) \\ -s(1-2\eta) & c(1-2\eta) & 1 \end{bmatrix}$$

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The first two columns are proportional

The prototype democratic model (suggestive but difficult to realize in a natural way): Fritzsch,Xhing

Assume that in first approximation:

$$m_{q,l} \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \xrightarrow[\text{diag}]{U} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$\bar{L}R$ Dirac

"Democratic"

$S_L x S_R$ perm. symmetry

For ν's, in the same basis:

$$m_\nu \sim a \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + b \cancel{\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}}$$

Both allowed by S_L

$L^T L$ Majorana

assume negligible (?)

In basis of m_e diagonal, imposing $s_{13} \sim 0$ (by hand)

$$U \sim \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix}$$

$\sin^2 2\theta_{atm} = 8/9$

Defect: solar more maximal than atmospheric

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Our general conclusion:

From the charged lepton sector:

a large s_{23} can easily be produced

example: lopsided models

U_e

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

m_e

but different orders for s_{12} and s_{13} is not simple

Still we have formulated a model where all mixings arise naturally from the charged lepton sector.

A set of U(1) charges garantees that m_ν is diagonal

The spectrum of one family is like in the 27 of E6

charged leptons

$$27 = 1 + 10 + 16 = 1 + (5 + \bar{5}) + (1 + \bar{5} + 10)$$

E6 SO(10) SU(5)

A see-saw mechanism involving the two sets of $\bar{5}$ leeds to the required zero determinant condition in m_e

The model works but requires a complicated setup of charges and flavons.

Note that it borrows the see-saw tricks from the neutrino model building

To make $m_\nu \sim 1$ a single U(1) is not enough:

$$m_\nu = \begin{bmatrix} \xi^{2p} & \xi^{p+1} & \xi^p \\ \xi^{p+1} & \xi^2 & \xi \\ \xi^p & \xi & 1 \end{bmatrix} m$$

In fact as $r \sim \xi^4 \sim 1/40$ then $\theta_{23} \sim \xi$ would be large

We need a flavour group

$$F = U(1)_{F_0} \times U(1)_{F_1} \times U(1)_{F_2} \times U(1)_{F_3}$$

F_i act on different light ν's

F_0 fixes quark and lepton hierarchies

	10_1	10_2	10_3	$\bar{5}^l_1$	$\bar{5}^l_2$	$\bar{5}^l_3$	5_H	$\bar{5}_H$	5_1	5_2	$\bar{5}_1$	$\bar{5}_2$
F_0	4	2	0	0	0	0	0	0	0	0	0	0
F_1	2	2	2	1	0	0	0	0	2	0	-2	0
F_2	2	2	2	0	1	0	0	0	2	0	-2	0
F_3	2	2	2	0	0	1	0	0	0	2	0	-2

The model is natural but cumbersome!

flavons →

	θ_0	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7
F_0	-1	0	0	0	0	0	0	0
F_1	0	-2	0	0	-3	0	0	-4
F_0	0	0	-2	0	0	-3	0	-4
F_0	0	0	0	-2	0	0	-3	-4

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We obtain a matrix of the form

$$m_e = \begin{bmatrix} O(\lambda^4) & O(\lambda^4) & O(\lambda^4) \\ x_{21}\lambda^2 & x_{22}\lambda^2 & O(\lambda^2) \\ x_{31} & x_{32} & O(1) \end{bmatrix} m$$

$m_e : m_\mu : m_\tau = \lambda^4 : \lambda^2 : 1$

det

We need $x_{21}x_{32} - x_{22}x_{31} = 0$ to guarantee an eigenvector of $m_e^+ m_e$ $[c, s, 0(\lambda^4)]$ with eigenvalue $0(\lambda^8)$: $s/c = -x_{31}/x_{32}$

- The hierarchy in the rows is from the $U(1)_{F0}$
- det=0 is arranged by a see-saw with dominance of a single heavy state in M^{-1} guaranteed by $U(1)_{F1} \times U(1)_{F2} \times U(1)_{F3}$

Note that $\theta_{13} \sim \lambda^4$ in this model

Summing up:

- ν masses very small -> Majorana ν's and see-saw mechanism
- ν masses are consistent with the standard way beyond the SM: SUSY and GUT's
- Recent exp progress:
  - Δm^2_{sol} went closer to Δm^2_{atm} → less hierarchy → $|m_3/m_2| \sim 6$
  - smaller upper limit on absolute mass: WMAP: $\Sigma m_\nu < 0.69$ eV
- Crucial issues:
  - LSND??
  - s_{13} small (how small?) disfavours anarchy
  - $s_{23} \sim$ maximal (too maximal?), $s_{12} \sim$ large not maximal disfavours inv. hierarchy
  - $0\nu\beta\beta$: near bound ? → degenerate ν's
    intermediate? → inverted hierarchy
    small ? → normal hierarchy → Looks simplest and fine
  - CP violation: still in the future

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