Note on Proton Rescattering

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Some old notes and slides from Jeon and Woijtech were incorporated into this note.

Abstract

We have studied the effects of proton rescattering on quasi-elastic events in the SciFi detector using the scattering model in Neut45 and earlier. Following a neutrino interaction within a nucleus, a recoil proton or neutron with significant momentum may be produced; always a proton for a CC-QE interaction. Before this nucleon leaves the nucleus, it may experience a further interaction. This report summarizes results from many studies and describes and compares the models used in the SciFi MA and Spectrum analysis. This document addresses several questions. How do we know that there is a non-negligible amount of proton rescattering occuring in the nucleus? What are the important details of the model we use? What are sufficient means to estimate the resulting measurement errors due to uncertainty in the amount of rescattering?

Introduction

In principle, proton rescattering affects how the recoil proton in quasi-elastic events are observed in the SciFi and SciBar detectors. Our current analyses use this information primarily to discriminate between quasi-elastic and non-quasi-elastic events in charged-current interactions.

If there is no rescattering of protons at all, the kinematics of the outgoing proton are completely determined by the interaction kinematics and Fermi motion effects, including Pauli blocking. The result is that, for protons with momentum above 600 MeV/c almost always come out within 20 degrees of what you would predict kinematically from the muon angle and momentum. This smearing is due to Fermi motion. Detector and reconstruction effects are not considered.

Rescattering of the proton inside the nucleus will cause a significant decrease in the outgoing momentum of the proton and will also change the scattered angle of the proton. This process is implemented in Neut in the form of a nucleon-nucleon (NN) cross-section based on the data and models presented in Bertini [Phys. Rev. C 6 (1972)]. Restricting the discussion to protons only, not all the protons will experience scattering. With this model, of the protons with more than 600 MeV/c momentum, approximately 40% experience a change in momentum as it leaves the nucleus. This change in momentum may cause the proton to exit the nucleus with a different angle or with a momentum that is below our three plane detection threshold. In some cases (according to the Neut MC) no proton will exit the nucleus at all.

0.0.1 Analysis from Neut interaction vectors only

The simplest and strongest observable is the distribution of the number of tracks. With proton rescattering a significant number of QE events that would have had two tracks will only have one because the proton will be produced below detection threshold. Comparing full rescattering (100% NN-XC) with less rescattering (80% NN-XC), there will be about 7% more QE-true two-track events with the latter. This is determined directly from Neut45 SciFi MC using cuts that approximate the SciFi detector response: muon momentum > 500 MeV/c, proton momentum > 600 MeV/c, QE-true. A full detector MC was not used for this estimate, but is described further below.

The second observable is how many two-track events migrate from the QE enriched sample ($\Delta\theta < 25$ degrees) to the nonQE enriched sample. Under the same conditions above, the QE-enriched sample is reduced by about 9% while the nonQE enriched sample is increased by about 9%. However, the QE enriched sample contains many more QE-true events; it is a good approximation to say that the entire 9% more nonQE obtained going from 80% NN to 100% NN came from the QE-enriched sample, while most of the events lost from the QE-enriched sample actually went to the 1-track sample.

The table shows the results for two sets of MC using the same number of MC files. The event comparison should not be exact because they are different MC samples. This only uses Neut vectors, no detector resolution effects are used, and these numbers are only for QE true, they do not attempt to describe any nonQE background. The cuts that separate the three sub-samples approximate the real detector efficiencies for SciFi.

sample	80% NN	100% NN	abs change	% change
1-track	176985	184980	-7995	-4.3%
QE-enhanced	105880	97463	8417	8.6%
nonQE-enhanced	9295	10141	-846	-8.3%
total	292160	292584	424	0.1%

For this study, I applied approximate selection cuts to divide into the three sub-samples. Since the most significant effect is that proton tracks fall below threshold, care should be taken that there is no Q^2 dependence to this threshold selection. The reweighting I propose is not directly energy or Q2 dependent; is this appropriate? For the Neut MC only, the answer is no.

0.0.2 Analysis with SciFi detector effects

When detector resolution effects are included, these distributions change significantly. The overall effect is smaller because of the increased smearing, and the proton detection threshold effect causes the nonQE-enhanced to be affected much more by protons dropping below threshold; see the different sign in the % change in nonQE sample. The following table shows the results for a full detector simulation using the SciFi MC event samples for the K2K-IIa configuration. Again, only QE events are selected here, based on the MC truth information. The percent change value is calculated AFTER normalizing the event samples because the statistics are somewhat small.

sample	80% NN	100% NN	abs change	% change
1-track	16749	16917	-168	-1.0%
QE-enhanced	2528	2420	108	4.5%
nonQE-enhanced	1092	1047	47	4.3%
total	norm	20384	N/A	N/A

Based on these results, it is possible to create a prescription to reweight the QE MC to account for this migration. If this is based exclusively on the Neut vector sample (no detector resolution smearing), then a satisfactory prescription is this. For a parameter of 0.90 (90% of the default NN cross-section), the following will reweight the default MC.

```
1tk.QEtrue x (1.0 - 0.010 x (1.0 - parameter) / 0.2)

2tkQE.QEtrue x (1.0 + 0.045 x (1.0 - parameter) / 0.2)

2tknonQE.QEtrue x (1.0 + 0.043 x (1.0 - parameter) / 0.2)
```

As with the neut vector analysis, even with full detector and selection effects, the 80% NN cross sections do not produce discernable Q^2 shape distortions due the combination of kinematics and selection. The reweighting used in this analysis need not have a functional dependence on Q2.

In addition to the above data tables, the change in the delta-theta distribution is shown in Fig 1.

One final comment: not including any proton rescattering at all causes a significant effect on the observed distributions in SciFi. In particular, the total number of two-track events in the MC would be approximately 20% too high; these excess events belong to the one-track sample.

Effect on the MA analysis

Because the MA analysis uses the systematic error pull-method to implement systematic effects, there are two things to consider: the specific implementation of the systematic error, and the central value and uncertainty used to add to the Chisquare of the fit, if any.

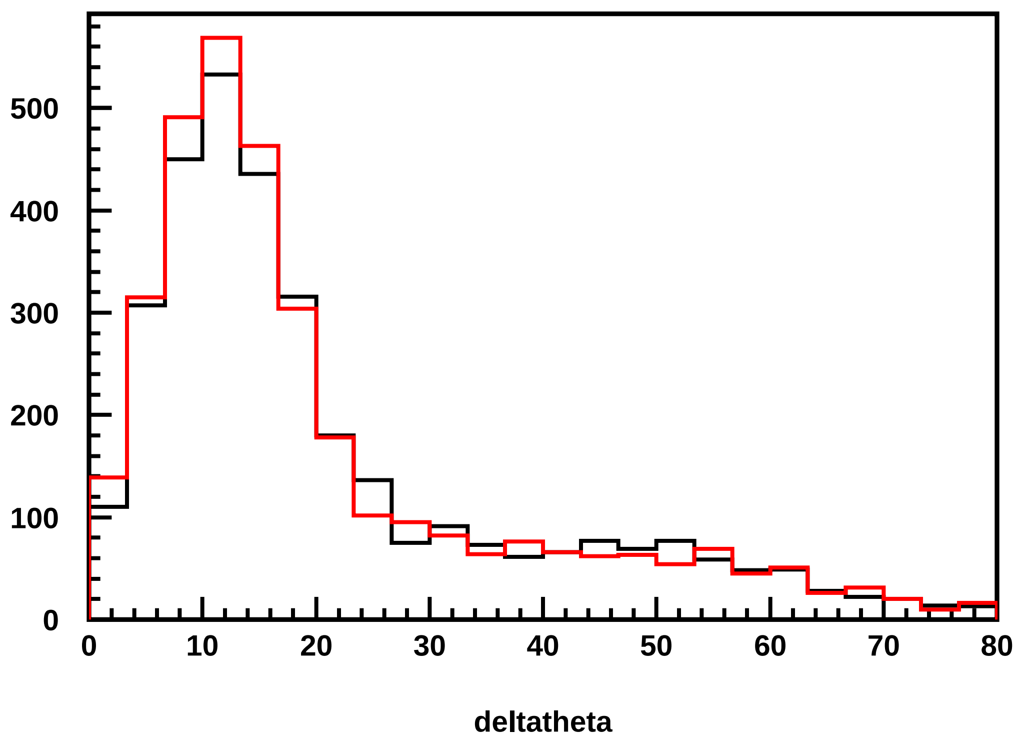

Figure 1: Black is default MC, Red is MC with 80% NN Cross-section. With a smaller cross-section, there are more events whose second track matches the QE prediction: most of these events would have been below detection threshold and in the 1-track sample with the full NN cross-section.

Basic result from the fit

Using the default Neut45 NN cross section and assuming a $\pm$ 20% uncertainty in this, the best fit value for MA is 1.198, with Chisquare 259.1. The best fit proton rescattering parameter is 1.06, in effect the best fit prefers 6% more scattering than the default. The contribution to the uncertainty in MA is about 2%.

Alternate results from the fit

Some comment on the evolution of this systematic error in the context of the MA analysis. An older analysis of the proton rescattering using Neut43, done by Jeon and carried through to my analyses throughout 2005, assumes a central value of 87% NN cross section and an error of 10% [See the section below "Comparison with electron scattering data"] gives an MA value of 1.16, chisquare of 259.7, and a fit value for this parameter of

0.90. In other words, the most significant effect, by far, is the change of the central value and error, and not the change in the parameterization. Using the new parameterization here, but retaining the old 87% central value gives MA = 1.17, Chisquare = 259.7, and parameter 0.89. Clearly, these simple parameterizations of proton rescattering have an insignificant effect on the fit-value of MA, relative to the other errors in this analysis. Further, it is not at all clear that the fit parameter for the rescaled NN cross-section contains unambiguous physics information about the parameter because the correlation between the other errors (known and unknown) appears to be very high.

Alternate implementations

In addition to implementing this error specifically from the MC samples, I have taken two other approaches to cross-check that the effect is negligible. I have implemented a two-track QE to two-track nonQE migration (for QE-true MC events) to complement the two-track to one-track migration already included in this analysis. This method is similar to the method used by SciFi and SciBar in the spectrum fit analysis. The result is MA=1.18, chisquare = 258, and the parameter that determines the fraction of events removed from the QE sample is 0.94; in effect there are 6% more events in the QE sample, taken from the nonQE sample. This fraction is applied the same to every kinematic bin used in the analysis.

The discussion in the paragraph above leads to an important point about this implementation. The dominant effect of less proton rescattering is to remove QE events from the two-track sample and migrate them to the one-track sample. The analysis already has a two-track to one-track migration implemented which is applied to all second tracks (proton or not, QE or not). The actual fit values for these two systematic error parameters are obviously correlated. Because of this, the implementation of the proton rescattering reweighting maybe should be considered a reweighting of the QE sample, with a much smaller effect on the 1-track and 2-track nonQE samples and the overall normalization.

Finally, since one of the effects that concerns us is how the QE and nonQE enhanced samples are separated, I studied the effect of changing the $\Delta\theta$ cut by $\pm$ 5 degrees, which had only a 1.5% effect on the fit value for MA. In some sense, this can be considered an independent confirmation that the fit value for MA is unaffected.

Comparison with Proton Scattering Data

We have in the past used the following citation for NN: Bertini, Phys. Rev. C 6 (1972) p. 631. Y. Hayato provided this citation in a proceedings of the NuInt conference.

More details of this calculation are provided by Woitech [Chris Walter captured a web page]. For protons specifically, the pp and pn cross-sections used (Bertini's) to model the scattering are given by the solid lines in Fig. 2.

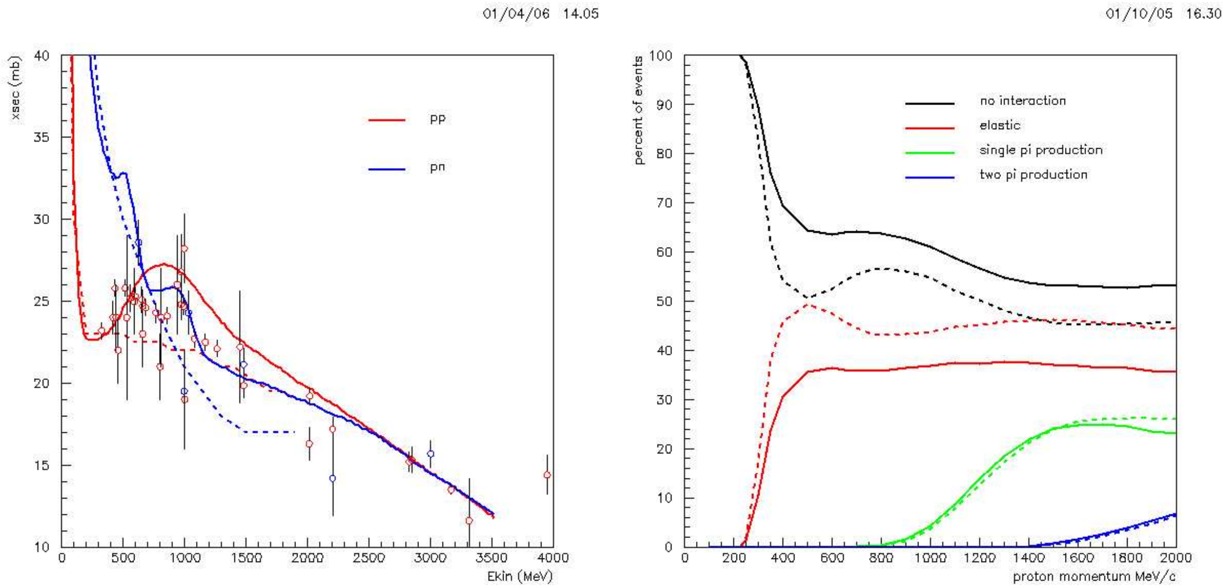

Figure 2: Plots by Woijtech. Solid lines are the cross sections of Bertini, dashed lines are from Dave (Casper?). Left: the pn and pp cross sections. Right: the fraction of protons that reinteract. The region of primary interest for SciFi is proton kinetic energy between 600 and 1000 MeV.

The more general consideration is that the pp and pn cross-sections are roughly 40 to 50 mb for incident protons at 1 GeV/c. About half of this (25 mb) is elastic scattering while the rest are inelastic, usually involving the production of a single pion in the final state. [I am not sure why these numbers, which Rik estimates from Bertini's paper are different than the ones Woitech gives in the plot Fig. 2.] The total cross-section on Oxygen and Aluminum around 1 GeV/c proton momentum is around 300 and 400 mb. [I am looking at R.F. Carlson Atomic and Nuclear Data Tables v63 (1993) p.96, but I think this information is available from a variety of sources.] These cross-sections increase with target nucleus as $A^{2/3}$ but are relatively constant for energies in this energy range.

The NN interaction cross sections are significantly higher for protons and pions with lower momentum, around 200 MeV/c. Experiments with low detection thresholds for second tracks may need to take these rescatterings even more seriously, especially for low Q2 interactions. In the case of SciFi, the momentum threshold for detecting a proton is around 600 MeV/c when we require three layers hit, where the NN cross sections are simple to model.

Woitech continues and presents his calculation for what scattering would be observed in a real nucleus, shown in Fig. 2. These large cross-sections for NN scattering contribute to the observation that 30% to 40% of the protons exiting the nucleus following the neutrino interaction experience some scattering on their way out.

Comparison with Electron Scattering Data

Members of the SciFi group made a simplified comparison of the effect of the Neut interaction model to data from an (e,e'p) experiment at the Yerevan accelerator. [K. Alanakyan, et al., Phys. Atomic Nucl. (translated from Yad. Fiz.) vol 61, no. 2 (1998) p. 256]. The specific goal was to look for proton rescattering effects in an experiment with essentially the same final state and similar energy. Eun Ju Jeon did much of the work for this comparison.

The procedure was to use the Neut interaction code and to generate electron interactions with Carbon at an energy of 1.94 GeV, to match the experimental setup of [Alanakyan 1998]. The comparison was the ratio of scattered protons at high angles to protons scattered at the center of the angle distribution around 66 degrees with respect to the beam direction. With nucleon rescattering effects, there is a significant tail to this distribution at high angles. We formed two such ratios: R(120/66) and R(140/66) representing scattering at two different places in the tail of this distribution; the objective is to estimate how much NN cross-section should be used in Neut.

Using data from [Alanakayan 1998], these ratios are R(120/66) = 0.0192 $\pm$ 0.0011 and R(140/66) = 0.0075 $\pm$ 0.0004. This comparison was originally done with Neut43 (shown in the final two columns) and later repeated with Neut45. The errors given are statistical, the Neut43 errors are similar, but slightly smaller.

Ratio	Data	no NFSI	100% NFSI	80% NFSI	Neut43 100%	Neut43 80%
R(120/66)	0.0192 $\pm$ 0.0011	< 0.0001	0.0364 $\pm$ 0.0021	0.0270 $\pm$ 0.0017	0.0233	0.0170
R(140/66)	0.0075 $\pm$ 0.0004	< 0.0001	0.0095 $\pm$ 0.0014	0.0074 $\pm$ 0.0009	0.0033	0.0019

The original conclusion from Jeon's study (around 2002) was that an appropriate center value to take for the proton rescattering pn and pp cross-sections is about 87% of the default Neut value, which gives agreement with the R(120/66) value from the data. The later study with Neut45 muddies this conclusion, and indicates that this effect might have a strong correlation with something else in the neutrino interaction model, or possibly the tail of the angular distribution requires a more sophisticated model than we currently use.

Other considerations for including this systematic error

This effect is presented here as a reweighting of QEtrue events in the Monte Carlo. However, the dominant effect is actually a migration of events between the two-track QE enhanced sample and the one-track sample, with a much smaller effect in absolute terms (simply because there are few QE events) on the nonQE enhanced sample. Because of the way nonQE background is included in our samples, in effect, only the two-track QE normalization is modified significantly when this parameter is changed.

Many of our analyses, especially the spectrum fit analyses for SciFi and SciBar, already include a 1-track to 2-track migration to account for detection efficiency uncertainty. This is clearly correlated with the proton rescattering effect presented here. Further, these analyses also include a nonQE/QE reweighting to account for uncertainty in the overall cross-section for these processes. Finally, in the SciFi and SciBar spectrum analyses, there is a migration between two-track QE and two-track nonQE samples.

While none of these reweightings are proton rescattering directly, it seems that a suitable linear combination of these reweightings offers the same treatment of the error due to rescattering effects, though it is in a way that makes it difficult to separate these effects. Likewise, it is not completely clear that an implementation of the proton rescattering effect described in this note and used in a fit such as the spectrum analysis or MA will give unambiguous information for how much proton rescattering there is in Oxygen.

Conclusion

A method for including proton rescattering in SciFi (and SciBar) data is described. Neut45 includes proton rescattering that affects roughly 40% of the protons above 600 MeV/c. We have presented a reweighting factor for the QE portion of the subsamples used in our analyses that show how an uncertainty of 20% in this cross-section (from 40% to 32% reinteraction) contributes on the order 4% changes in each two-track sub-sample.