Inclusive Quasielastic ν Reaction in ^{16}O

S. K. Singh and M. Sajjad Athar[a]*

[a] Department of Physics, Aligarh Muslim University,
Aligarh-202 002, India

We calculate the inclusive quasielastic ν_μ cross sections in ^{16}O. The calculations are done in the Local Density Approximation(LDA) and take into account Pauli blocking, Fermi motion and the effect of renormalization of weak coupling constants in the nuclear medium.

We study the inclusive reation

$$\nu_l +^{16} O \to l^- + X, l = e, \mu \quad and \tag{1}$$

the matrix element for the basic ν-nucelon reaction

$$\nu_l(k) + n(p) \to l^-(k') + p(p') \tag{2}$$

is given by [1]:

$$T = \frac{G_F}{\sqrt{2}} \cos\theta_c l_\mu J^\mu, \tag{3}$$

where

$$l_\mu = \bar{u}(k')\gamma_\mu(1-\gamma_5)u(k) \tag{4}$$

$$\begin{aligned} J_\mu \\ &= \bar{u}(p')\left[F_1^V(q^2)\gamma_\mu + F_2^V(q^2) i\sigma_{\mu\nu}\frac{q^\nu}{2M} \right. \\ &\left. + F_A^V(q^2)\gamma_\mu\gamma_5 + F_p^V(q^2)q_\mu\gamma_5\right] u(p) \end{aligned} \tag{5}$$

Here $q = k - k'$ is the momentum transfer and $F_1^V(q^2)$ and $F_2^V(q^2)$ are isovector vector form factors which are determined in terms of electromagnetic form factors $F_{1,2}(q^2)$ for the nucleon, assuming conserved vector current (CVC) hypothesis and are given in terms of experimentally determined Sach's electric and magnetic form factors $G_E(q^2)$ and $G_M(q^2)$:

$$F_1^V(q^2) = \left(1 - \frac{q^2}{4M}\right)^{-1}\left[G_E^V(q^2) - \frac{q^2}{4M^2}G_M^V(q^2)\right]$$

$$F_2^V(q^2) = \left(1 - \frac{q^2}{4M}\right)^{-1}\left[G_M^V(q^2) - G_E^V(q^2)\right]$$

where

$$G_E^V(q^2) = \frac{1}{\left(1-\frac{q^2}{M_V^2}\right)^2}\left[1 - \frac{q^2\mu_n}{4M^2}\right],$$

$$G_M^V(q^2) = \frac{1+\mu_p-\mu_n}{\left(1-\frac{q^2}{M_V^2}\right)^2}$$

with $M_V^2 = 0.71 GeV^2$, μ_p= $1.792847\mu_N$ and μ_n= -$1.913042\mu_N$.

$F_A^V(q^2)$ is the axial vector form factor given by[2]:

$$F_A^V(q^2) = \frac{F_A(0)}{\left(1-\frac{q^2}{M_A^2}\right)^2} \tag{6}$$

where M_A is the dipole mass taken to be 1.03GeV and $F_A(0)$=1.26.

The pseudoscalar form factor $F_p^V(q^2)$ is given by

$$F_p^V(q^2) = \frac{2MF_A^V(q^2)}{M_\pi^2 - q^2} \tag{7}$$

The differential scattering cross section $d\sigma/d\Omega dE'$ for the reaction described in Eq. (2) is given by

$$\begin{aligned} &\frac{d\sigma}{d\Omega dE'} \\ &= \frac{1}{4\pi^2}\frac{E_l}{E_\nu}\frac{M_nM_p}{E_nE_p}\sum\bar{\sum}|T|^2\delta(E_\nu+E_n-E_l-E_n) \end{aligned} \tag{8}$$

---
*sk_singh@rediffmail.com

where $\bar{\sum}\sum|T|^2$ is the square modulus of the transition amplitude given in Eq. (3), summed over the initial and the final spins of the nucleons.

When the reaction takes place in a nucleus, the neutrino scatters from a neutron which is not free and the expression given in Eq. (8) has to be modified because of the nuclear medium. Such medium modification effects have been discussed in our previous work [1,2] using local density approximation where details of the model used here are discussed. In a nucleus, the neutrino scatters from a nucleon whose local density in the medium is $\rho_N(r)$ correspondig to the local Fermi momentum $p_N = \left[3\pi^2\rho_N(r)\right]^{\frac{1}{3}}$, $N = n, p$. The neutrons and protons are in the nucleus, where their momenta are constrained to satisfy the Pauli principle, i.e., $p < p_{F_n}$ and $p'(=|\mathbf{p}+\mathbf{q}|) > p_{F_p}$, p_{F_n} and p_{F_p} being the local Fermi momenta of neutrons and protons at the interaction point in the nucleus. Moreover, the Q-value of the reaction and the Coulomb energy of the outgoing lepton produced inside the nucleus has to be taken into account.

These modifications are incorporated by replacing the energy conserving δ-function in equation(8) by $Im\bar{U}(q_0,\vec{q})$, where $U(q_0,\vec{q})$ is the Lindhard function corresponding to the particle-hole (ph) excitation shown in Fig.1 and is given by:

$$U(q_0,\mathbf{q})$$

$$= \int \frac{d\mathbf{p}}{(2\pi)^3}\frac{n_n(\mathbf{p})\left[1-n_p(\mathbf{p}+\mathbf{q})\right]}{q_0+E_n(\mathbf{p})+E_p(\mathbf{p}+\mathbf{q})+i\epsilon}\frac{M^2}{E_nE_p} \quad (9)$$

with $q_0 = E_{\nu_\mu} - E_\mu - Q - V_c(\mathbf{r})$, where Q is the Q value of a reaction. In an inclusive reaction there is no unique value for Q, as many transitions to the final states are involved. The Q value corresponding to the ground state transitions to the final state is taken for calculations. $V_c(\mathbf{r})$ is the Coulomb energy of the outgoing lepton in the final nucleus which is given by:

$$V_c(r)$$

$$= ZZ'\alpha 4\pi\left[\frac{1}{r}\int_0^r \frac{\rho_p(r')}{Z}r'^2 dr' + \int_r^\infty \frac{\rho_p(r')}{Z}r' dr'\right], \quad (10)$$

where Z' is the charge of the final lepton produced in the reaction.

The imaginary part of the Lindhard function defined in equation (9) is evaluated to give:

$$Im\bar{U}(q_0,\vec{q}) = -\frac{1}{2\pi}\frac{M_pM_n}{|\vec{q}|}\left[E_{F_1} - A\right] \quad (11)$$

$$q^2 < 0, \quad E_{F_2} - q_0 < E_{F_1}$$

and

$$\frac{-q_0 + |\vec{q}|\sqrt{1-\frac{4M^2}{q^2}}}{2} < E_{F_1}$$

where,$E_{F_1} = \sqrt{p_{F_n}^2 + M_n^2}$,$E_{F_2} = \sqrt{p_{F_p}^2 + M_p^2}$ and

$$A = Max\left[M_n, \;\; E_{F_2} - q_0, \;\; \frac{-q_0 + |\vec{q}|\sqrt{1-\frac{4M^2}{q^2}}}{2}\right]$$

Otherwise $Im\bar{U}(q_0,\vec{q}) = 0$.

Furthermore, in a nucleus the weak-coupling constants like F_A^V, F_2^V, etc. are renormalized due to the presence of strongly interacting nucleons in the nuclear medium. This renormalization is taken into account by generalizing the diagram shown in Fig.1 to Fig.2 for calculating the imaginary part of the Lindhard function defined in Eq. (11). This diagram simulates the effect of nucleon-nucleon strong force on the weak vertex.

The nucleon-nucleon potential is taken to be of the form

$$V(q) =$$

$$\frac{f^2}{m_\pi^2}\left[V_l\hat{q}_i\hat{q}_j + V_t(\delta_{ij} - \hat{q}_i\hat{q}_j)\right]\sigma_i(1)\sigma_j(2)\tau(1)\tau(2), \quad (12)$$

where V_l and V_t are the strengths of the potential in the longitudinal and transverse channels. The following forms corresponding to π and ρ exchanges have been used for the potentials V_l and V_t

$$V_l(q) = \frac{f^2}{\mu^2}\left[\frac{q^2}{-q^2+\mu^2}\left(\frac{\Lambda^2-\mu^2}{Lambda^2-q^2}\right)^2 + g'\right]$$

$$V_t(q) = \frac{f^2}{\mu^2}\left[\frac{q^2}{-q^2+m_\rho^2}C_\rho\left(\frac{\Lambda^2-m_\rho^2}{\Lambda^2-q^2}\right)^2 + g'\right] \quad (13)$$

$\Lambda = 1.3GeV$, $C_\rho = 2$, $\Lambda_\rho = 2.5$, μ and m_ρ are the pion and ρ masses, and g' is the Landau-Migdal parameter taken to be 0.7 in the present calculations [3]. The Δh interactions are obtained from equations (12 & 13) by replacing $\sigma \rightarrow \mathbf{S}, \tau \rightarrow \mathbf{T}$ and $f \rightarrow f^* = 2.5f$ for any Δ which replaces a nucleon, where $\mathbf{S}$ and $\mathbf{T}$ are spin and isospin transition operators. The effect of calculating the diagram shown in Fig.2 is that the quardratic terms in $|T^2|$ such as F_2F_A,F_A^2, etc. are modified as follows [4]:

$$F_A^2 \rightarrow \left[\frac{1}{3}\frac{1}{|1-UV_l|^2} + \frac{2}{3}\frac{1}{|1-UV_t|^2}\right]$$

$$(F_2^2, F_2F_A) \rightarrow (F_2^2, F_2F_A)\frac{1}{|1-UV_t|^2}$$

where,
$U = U_N + U_\Delta$
U_N = Linhard function for particle-hole excitation.
U_Δ = Lindhard function for Δ-hole excitation.
V_l , V_t = Potential due to π, ρ exchange.

The expressions for U_N and U_Δ are given in refs.[5,6].

With these modifications the differential cross section is written as:

$$\begin{aligned}\frac{d\sigma}{d\Omega dE'} &= -\frac{G_F^2 Cos^2\theta_c}{\pi^2}\int_{r_{min}}^{r_{max}} r^2 dr p_l^2 \times \\ &\frac{1}{E_\nu E_l}\bar{\sum}\sum |T|^2 \times \\ &Im\bar{U}[E_\nu - E_l - Q - V_c(\vec{r}), \vec{q}] \\ &\times\Theta[E_l + V_c(r) - m_l], \qquad (14)\end{aligned}$$

The value of ρ_n used in our calculation is obtained from the charge density of the nucleus $\rho(r)$ by using the relation $\rho_n(r) = \frac{(A-Z)}{A}\rho(r)$ and $\rho_p(r) = \frac{Z}{A}\rho(r)$. For ^{16}O nucleus we have used harmonic oscillator(H.O) density distribution [7] given by

$$\rho(r) = \rho_0\left(1 + c_1\left(\frac{r}{c_2}\right)^2\right) exp\left(-\left(\frac{r}{c_2}\right)^2\right) \quad (15)$$

where $c_1 = 1.805$, $c_2 = 1.517fm$.

The numerical results for $\frac{d^2\sigma}{4\pi d(cos\theta_\mu)dE_\mu}$, $\frac{d\sigma}{dE_\mu}$ and $\frac{d\sigma}{4\pi d(cos\theta_\mu)}$ are presented in figures.3, 4 and 5. We have also presented the results for $d\sigma/dq^2$ and the total cross sections $\sigma(E)$ in figures 6 & 7. The model presented in this paper has been successful in describing the nuclear medium effects in muon capture and the quasielastic neutrino reactions in the energy region of the LSND reaction [3,4]. The numerical results presented in this paper in the energy region of atmospheric neutrinos are more reliable as our method is expected to give better results at higher energies.

The work presented here has been partially done in collaboration with E.Oset of Univ. of Valencia, Spain and we will like to thank him for his interest and help in this work.

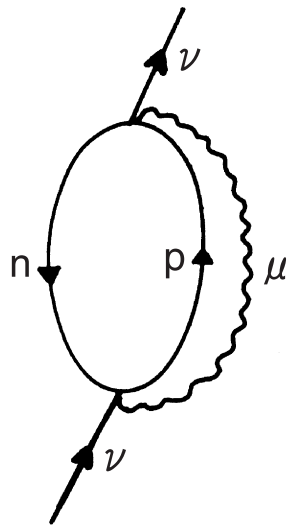

Figure 1. Feynman diagram for the neutrino self energy related to $\nu_\mu + n \rightarrow \mu^- + p$.

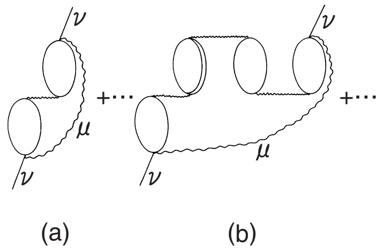

Figure 2. Many-body Feynman diagrams accounting for the medium polarization in the spin-isospin channel driven by $\nu_\mu + n \rightarrow \mu^- + p$ transition.

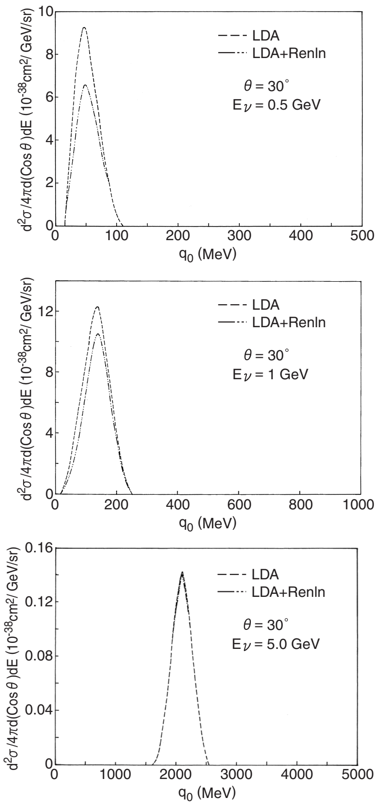

Figure 3. $\frac{d^2\sigma}{4\pi d(cos\theta_\mu)dE_\mu}$ vs $q_0(= E_{\nu_\mu} - E_\mu)$ at θ_μ=30^0 for E_{ν_μ} = 0.5GeV(Fig.3a), 1.0GeV(Fig.3b) and 5.0GeV(Fig.3c).

REFERENCES

1. M.Sajjad Athar and S.K.Singh, Phys. Rev.C61 028501(2000)
2. S.K.Singh and E.Oset, Phys. Rev.C48 12461(1993); T.S.Kosmas and E.Oset, Phys.Rev.C53 1409(1996).
3. N.C.Mukhopadhay, H.C.Chiang, S.K.Singh and E.Oset, Phys. Letts. B434 7(1998); N.C.Mukhopadhay, S.K.Singh and E.Oset, Phys.Rev. C57 2686(1998).
4. S.K.Singh and E.Oset, Nucl.Phys.A542 587(1992).
5. E.Oset, D. Strottman, H.Toki and J.Navarro, Phys. Rev.C48 2395(1993); E.Oset and M.Rho, Phys.Rev.Letts.42 47(1979).
6. E.Oset, P.Fernandez de Cordoba, L.L.Salcedo and R.Brockmann, Phys.Rep.188 79(1990).
7. C.W.de Jager, H.de Vries and C.de Vries Atomic Data & Nucl. Data Tables14 479(1974).

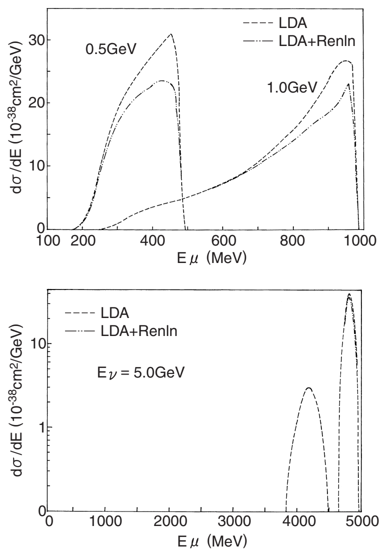

Figure 4. $\frac{d\sigma}{dE_\mu}$ vs E_μ for $E_\nu = 0.5$GeV, 1.0GeV(Fig.4a) and 5.0GeV(Fig.4b).

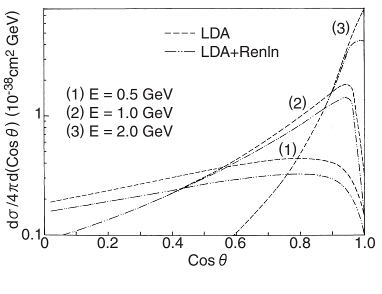

Figure 5. $\frac{d\sigma}{4\pi d(cos\theta_\mu)}$ vs cosθ_μ for $E_\nu = 0.5$GeV, 1GeV and 2GeV.

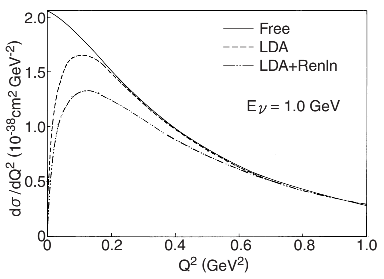

Figure 6. $d\sigma/dq^2$(per nucleon) for ^{16}O as a function of $Q^2(= -q^2)$ in the free case, in the local density approximation(LDA) without and (LDAR)with renormalization effects for $E_\nu = 1.0$GeV.

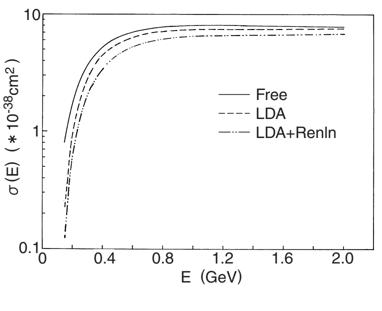

Figure 7. $\sigma(E)$ vs E, for ν_μ reactions in ^{16}O.