Fermi Gas Model

Gerald A. Miller [a] *

[a]Department of Physics, University of Washington
Seattle, Washington 98195-1560, USA

A few comments about the application of the Fermi Gas Model to neutrino interactions with nuclei are made as part of a panel discussion.

1. INTRODUCTION

The Fermi Gas Model is the simplest mean field model of nuclear physics. One treats the nucleus as non-interacting nuclear matter in the limit that the radius of the nucleus becomes infinite. The nucleon occupation number $n(k)$ is a simple step function: $n(k) = \theta(k_F - k)$. The A-dependence of finite nuclei is modeled by choosing the Fermi momentum k_F, and an average binding energy to represent the physics of a given target. The principle advantage of this model is that the Pauli principle is treated correctly and that one can obtain the neutrino-nuclear cross section in terms of invariant structure functions. It may give a good accounting for inclusive cross sections measured at quasi-elastic kinematics[1].

My main message here is that this procedure does not provide enough input regarding nuclear structure to describe the electron scattering reactions: $(e,e'),(e,e,'p)$ and (e,e',π) for general kinematics. Therefore there is no reason to believe that such a simple procedure should be very accurate in describing neutrino-nuclear interactions.

2. FERMI GAS MODEL

The first and simplest remark is that target nucleus such as ^{16}O is not a Fermi gas. This nucleus has a finite size and its momentum distribution can not be described in terms of only a single parameter. Indeed, the nucleon momentum distribution, as calculated from mean field theory does not closely resemble the parabolic shape of the Fermi gas model. See for example, Fig. 2 of Ref. [2]. The distribution of a finite nucleus falls off rapidly like a Gaussian, and therefore looks narrower and is higher at its peak. However, in any case the peak is narrow. If the kinematics of the measurement involves integration over the width of the peak, the computed cross sections should be reasonable estimates. For a detailed study of phenomena beyond the scope of of Fermi gas models see Ref [3]. For a review of related calculations, see Ref [4]. Some relevant future work will also appear[5].

2.1. Spectral Function and Correlations

One might think that one could simply take a function $n(k)$ from an improved theoretical calculation. But this would not be sufficient, for non-quasieleastic kinematics. The correct way to compute the cross section for the (e,e') reaction depends on the use of the spectral function defined by a ground state matrix element involving the complete Hamiltonian H:

$$P(\mathbf{p},E) = \langle\Psi|a^\dagger(\mathbf{p})\delta(E-H)a(\mathbf{p})|\Psi\rangle, \tag{1}$$

which gives the probability of removing a nucleon of momentum $\mathbf{p}$ while leaving the nucleus with an excitation energy E. The relation with the occupation number is

$$n(\mathbf{p}) = \int dE\; P(\mathbf{p},E). \tag{2}$$

As a result of the correlations between nucleons, the function $n(\mathbf{p})$ has a long tail in momentum space. See, for example, Fig. 1 of Ref. [6]. This shows that n is not a step function.

*This work is partially supported by the USDOE.

2.2. Cross section

The spectral function is related to the cross section by the following schematic formula [7]:

$$d\sigma \sim \int d^3p_{A-1}dE\ P(\mathbf{p}_{A-1}, E)|\mathcal{M}|^2 \times \\ \delta^{(3)}(\mathbf{p}_\nu - (\mathbf{p}_\mu + \mathbf{p} + \mathbf{p_{A-1}})) \times \\ \delta(E_\nu - (E_\mu + \sqrt{p^2 + M_p^2} + B)), \qquad (3)$$

where $\mathbf{p}$ represents the momentum of a knocked out nucleon and B represents the binding energy difference between the target and the residual nucleus of $A-1$ nucleons. This treatment of conservation of energy and momentum differs from one in use at KEK for which $E_\nu + \sqrt{p_{A-1}^2 + M_p^2} = E_\mu + \sqrt{p^2 + M_p^2} + B$. The result (3) is more appropriate if one goes beyond the simplest Fermi gas.

But there are other difficulties which must be handled. The first of these involves final state interactions, which is important for computing the cross section for the $(\nu, \mu p)$ reaction.

3. FINAL STATE INTERACTIONS

If one is computing the cross section for the reaction $(e, e'p)$ one needs to include the effects of the interactions between the outgoing proton and the residual nucleus. It is typical to use an optical potential to describe this interaction. However, this object, determined by elastic proton-nucleus scattering, does not determine the wave function of the outgoing proton for positions inside the nucleus. This leads to an uncertainty in computing the cross section. The minimum uncertainty one can expect due to this source is about 5%.

Another difficulty is that without knowing the properties of the residual nuclear final states, one doesn't really know if one should use the optical potential at all.

But there are other sources of uncertainty.

4. AXIAL VECTOR FORM FACTOR

To calculate the ν-nucleon cross section one needs to know both the axial vector form factor F_A and the pseudoscalar form factor F_P. PCAC relates these two as

$$F_P(q) = \frac{2MF_A(q)}{(m_\pi^2 - q^2)}, \qquad (4)$$

but one needs to know F_A. This is usually parameterized as a dipole form factor. In addition to the uncertainties in the dipole parameter, there is a larger question of whether or not the dipole formula applies.

Indeed, recent work on the proton electric form factor shows that this does not obey a dipole shape[8],[9] for momentum transfers $Q^2 > 1\mathrm{GeV/c}^2$. For one explanation of this interesting phenomenon see Ref. [10].

The relevance to neutrino-nucleon interactions is that one does not really know the nucleon form factors to very high accuracy.

5. SUMMARY

The Fermi gas model may provide a reasonable estimate of the ν, μ cross section for quasi-elastic kinematics. One caution is that this model is not well tested. A detailed description of the procedure has been presented in the lecture notes of Seki & Nakamura[11].

Here is a list of comments.

- For detailed calculations one really should use a spectral function which is computed especially for the chosen target nucleus, here ^{16}O.

- There is considerable gain in uncertainty when there is a gain in exclusivity. Computing the (ν, μ, p) cross section is considerably more difficult than the one for (ν, μ).

- There are other uncertainties to due lack of knowledge of form factors.

- One may be able to increase accuracy by checking that any theory procedure used for neutrino scattering actually works when used for electron scattering.

Acknowledgements

I thank R. Seki, W. C. Haxton and K. Langacke for useful discussions.

REFERENCES

1. R. A. Smith and E. J. Moniz, Nucl. Phys. B **43**, 605 (1972) [Erratum-ibid. B **101**, 547 (1975)].
2. J. R. Smith and G. A. Miller, Phys. Rev. C **65**, 055206 (2002) [arXiv:nucl-th/0202016].
3. J. Engel, E. Kolbe, K. Langanke and P. Vogel, Phys. Rev. D **48**, 3048 (1993) [arXiv:nucl-th/9304017].
4. P. Vogel, arXiv:nucl-th/9901027.
5. E. Kolbe, W. C. Haxton, K. Langanke and P. Vogel, "Modeling neutrino-induced reactions in different energy ranges" to appear
6. C. Ciofi degli Atti and S. Simula, Phys. Rev. C **53**, 1689 (1996) [arXiv:nucl-th/9507024].
7. O. Benhar, A. Fabrocini, S. Fantoni, G. A. Miller, V. R. Pandharipande and I. Sick, Phys. Rev. C **44**, 2328 (1991).
8. M. K. Jones *et al.*, Phys. Rev. Lett. **84**, 1398 (2000)
9. O. Gayou *et al.*, Phys. Rev. Lett. **88**, 092301 (2002).
10. G. A. Miller and M. R. Frank, nucl-th/0201021 to appear Phys. Rev. C.
11. "Lecture Notes on Intermediate-Energy Neutrino-Nucleus Reactions" R. Seki and H. Nakamura