Convener's and Spectrum Meeting
May 15, 2002
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Official plots and numbers for Oscillation analysis

May, 2002

Likelihood

$$L_{tot} = L_{norm}(f) \cdot L_{shape}(f) \cdot L_{syst}(f)$$

systematics parameters

$$f = (f_{\Phi}, f_{nQE}, f_{\mathrm{F/N}}, f_{\varepsilon sk}, f_{Esk}, f_{n6}, f_{n11})$$

spec

nQE/QE

Far/near

ε_{SK}(efficiency)

SK Enegry scale

Overall Norm Error
For Jun99 and Nov99-

Systematic parameters also treated as fitting parameter.

Normalization term

$$L_{norm} = Poisson(N_{obs}, N_{\exp}(f))$$

$$N_{obs} = N_{6obs} + N_{11obs} = 1 + 55 = 56$$

$$N_{exp}(f) = N_{6exp}(f, \Delta m^2, \sin^2 2\theta) + N_{11exp}(f, \Delta m^2, \sin^2 2\theta)$$

$$N_{6\exp}(f, \Delta m^2, \sin^2 2\theta) = N_{6\exp}(f_{n6}, \Delta m^2, \sin^2 2\theta)$$
$$= 4.55 + f_{n6} \quad \text{(when no oscillation)}$$

N_{11exp}

$$N_{11\exp} = N_{11\exp}(f_{\Phi}, f_{nQE}, f_{F/N}, f_{n11}, \Delta m^2, \sin^2 2\theta)$$

$$= \alpha \cdot \frac{N_{SK}^{MC}(f, P_{osc})}{N_{KT}^{MC}(f)}$$

Cancellation

$$= \alpha \cdot (1 + f_{n11}) \cdot \frac{\sum_{i,j} f_{\Phi i} f_{F/Ni} \cdot \Phi_{SK}^{MC}(E_i) P(\Delta m^2, \sin^2 2\theta) \cdot (f_j \sigma_{ij}) \cdot \varepsilon_{ij}^{SK}}{\sum_{i,j} f_{\Phi i} \cdot \Phi_{KT}^{MC}(E_i) \cdot (f_j \sigma_{ij}) \cdot \varepsilon_{ij}^{KT}}$$

i :energy, j :mode

α: normalization factor

Shape term for FC 1Rμ

$$L_{shape} \equiv \prod_{i=1}^{29} P\left((f_{Esk} \cdot E_i), \Delta m^2, \sin^2 2\theta, f\right)$$

- ***P***: binned (50MeV) reconstructed Eν distribution for 1Rμ gen'ed using MC
- Energy scale systematic parameter applied to the observed E_ν^{rec}

Total 29 events for **Nov99~** data

Systematic constraint term

$$
\begin{aligned}
L_{syst} \equiv \quad & \exp\left(-\Delta f^{T}_{\Phi,nQE}\cdot M^{-1}_{FD}\cdot \Delta f_{\Phi,nQE}/2\right) \\
& \times\exp\left(-\Delta f^{T}_{F/N}\cdot M^{-1}_{F/N}\cdot \Delta f_{F/N}/2\right) \\
& \times\exp\left(-\Delta f^{T}_{\varepsilon SK}\cdot M^{-1}_{\varepsilon SK}\cdot \Delta f_{\varepsilon SK}/2\right) \\
& \times\exp\left(-f^{2}_{n6}/2\sigma^{2}_{n6}\right)\cdot\exp\left(-f^{2}_{n11}/2\sigma^{2}_{n11}\right) \\
& \times\exp\left(-\Delta f^{2}_{Esk}/2\sigma^{2}_{Esk}\right)
\end{aligned}
\qquad \Delta f \equiv f - 1
$$

M_{FD}, M_{π}, M_{SK}: error matrix of syst. errors.

σ_{n6}: overall norm. error on N_{6exp} (=+0.80–0.68 evts)

σ_{n11}:overall norm. err. on N_{11exp} (=5.34%)

σ_{Esk}: SK Enery scale error (3%)

(described later)

Summary of Nsk for Jun99

4.55evt.
+0.80evt
–0.68evt

Use this error as constant
even for oscillated case

PIMON error dominant
(PMT saturation etc)
→ uncorrelated w/ Nov99~

Syst. error (event)

	Old	New	
Stat	0.07	0.07	1.5%
KT	0.20	0.20	4.4%
SK	0.14	0.14	3.1%
Flux	+0.26 –0.32	0.42	9.3%
F/N	+0.62 –0.45	+0.62 –0.45	13.6% 9.9%
NC/CC	0.01	0.01	
nQE/QE	+0.02 –0.04	+0.07 –0.11	1.6% 2.4%
CT	0.04	0.04	0.9%
Total	+0.72 –0.61	+0.80 –0.68	17.6% 14.9%

Overall normalization error on Nsk for Nov99~

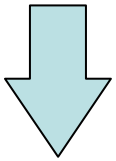
Central Value
76.05evts

KT: dominated
by FV error
SK: also.

Errors

	(Event)	
Stat	0.28	0.37%
KT	3.32	4.37%
SK	2.28	3.00%
Flux	+2.81 –2.59	
F/N	+4.26 –5.55	
NC/CC	+0.15 –0.23	
nQE/QE	+0.38 –0.61	
CT	0.46	0.60%
Total	+6.53 –7.37	

Take errors not considered in matrix



5.34%

Central values for syst. params

f_{Φ} and f_{nQE} : fitted center for FD data w/ M_A wgt'ed MC

1	0-500	1.3136
2	500-750	1.0208
3	750-1000	1.0071
4	1000-1500	1.0000
5	1500-2000	0.95487
6	2000-2500	0.96180
7	2500-3000	1.1830
8	3000-	1.0726
9	QE	1.0000
10	non-QE	0.92856

Center

$f_{\varepsilon\mathrm{SK}}$ and $f_{\mathrm{F/N}}$ **=1** f_{n6} and f_{n11} **=0**

Error matrix from FD

"sqrt" of error matrix

********** SQRT(FD ERROR MATRIX) (%) **********

48.8876	-11.4455	6.5955	0.9263	2.1190	-4.1593	5.1186	-15.0333
-11.4455	11.9164	-2.9850	2.6796	0.9894	2.2561	2.9715	-7.5033
6.5955	-2.9850	9.1652	4.6797	-0.8666	4.5497	2.0928	-4.3128
0.9263	2.6796	4.6797	7.0711	-2.1213	7.6420	4.9295	-3.4496
2.1190	0.9894	-0.8666	-2.1213	8.4321	-7.5829	8.1670	-3.4059
-4.1593	2.2561	4.5497	7.6420	-7.5829	18.7617	-11.1803	-3.0757
5.1186	2.9715	2.0928	4.9295	8.1670	-11.1803	19.8746	-6.7528
-15.0333	-7.5033	-4.3128	-3.4496	-3.4059	-3.0757	-6.7528	20.2731
f1	f2	f3	f5	f6	f7	f8	nQE/QE

Enlarged

F/N & SK matrix

********** SQRT(FAR/NEAR ERROR MATRIX) (%) **********

0-0.5	0.5-1	1-1.5	1.5-2	2-2.5	2.5-
2.5696	-1.6432	0.0000	0.0000	0.0000	0.0000
-1.6432	4.2591	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	6.5322	2.3973	-1.7861	-4.0187
0.0000	0.0000	2.3973	10.4307	9.5315	4.9528
0.0000	0.0000	-1.7861	9.5315	11.0833	7.2229
0.0000	0.0000	-4.0187	4.9528	7.2229	12.1552

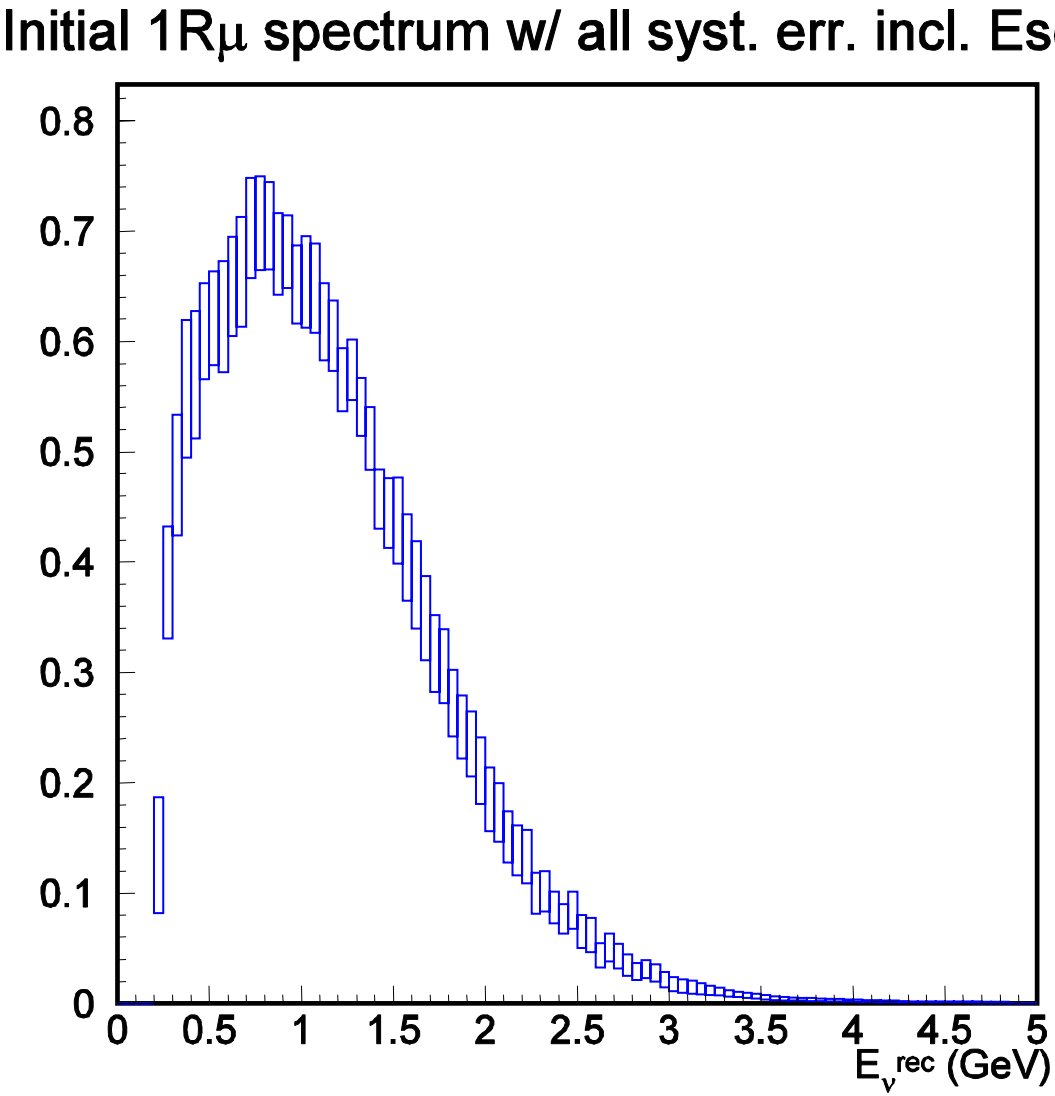
********** SQRT(SK ERROR MATRIX) (%) **********

0-0.5	0.5-1	1-1.5	1.5-2	2-2.5	2.5-	Escale
8.7000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	4.3000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	8.9000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	10.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	9.8000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	9.9000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	3.0000

Summary of systematic errors on total # of events w/o osc.

Center: 80.1evts		(evts)	(%)
Jun99 (4.55evts)	Total	+0.8 –0.7	+1.00% –0.85%
Nov99~ (75.5evts)	Spectrum	+0.4 –0.5	+0.56% –0.63%
	nQE/QE	+0.4 –0.9	+0.47% –1.14%
	Far/Near	+4.0 –4.1	+4.93% –5.01%
	Norm	4.0	5.03%
	Subtotal	+6.1 –5.3	+7.62% –6.66%
Total		**+6.2** **–5.4**	**+7.67%** **–6.72%**

Exp'ed E_ν^{rec} spectrum @ SK for 1Rμ estimated by FD measurements

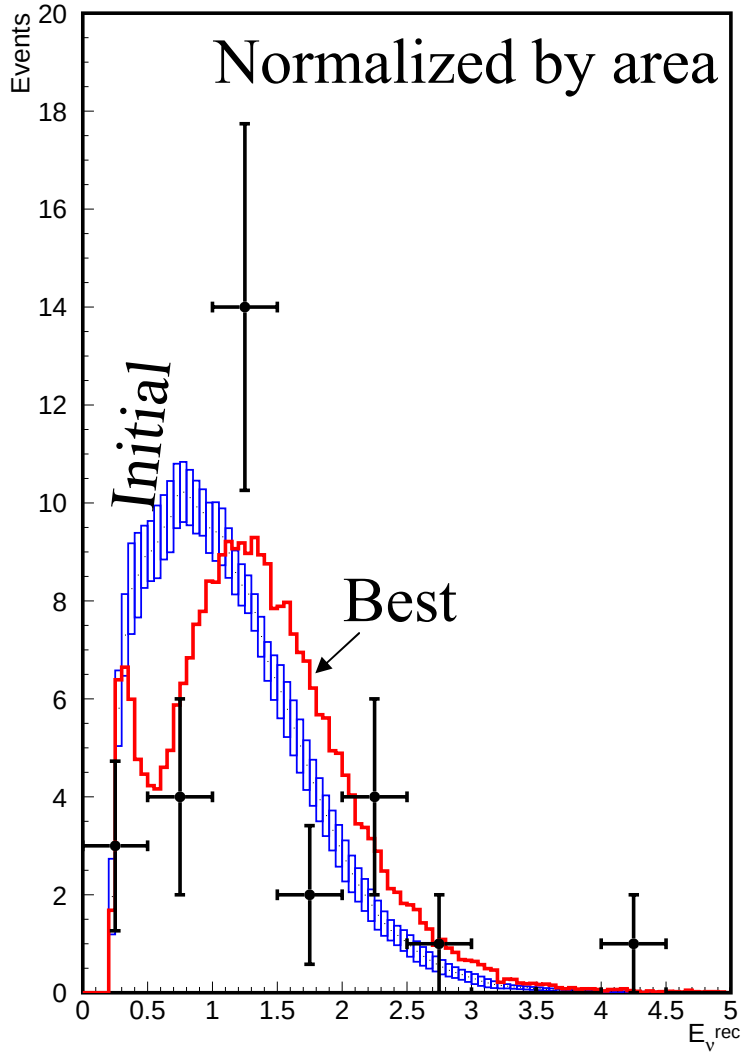


Summary of best fit points

		$\sin^2 2\theta$	Δm^2
Global	1Rμ shape only	1.09	3.0
	Shape + Norm.	1.03	2.8
Phys. Region	1Rμ shape only	1.00	3.0
	Shape + Norm.	1.00	2.8

$(10^{-3}eV^2)$

Best fit 1Rμ spectrum & Nsk



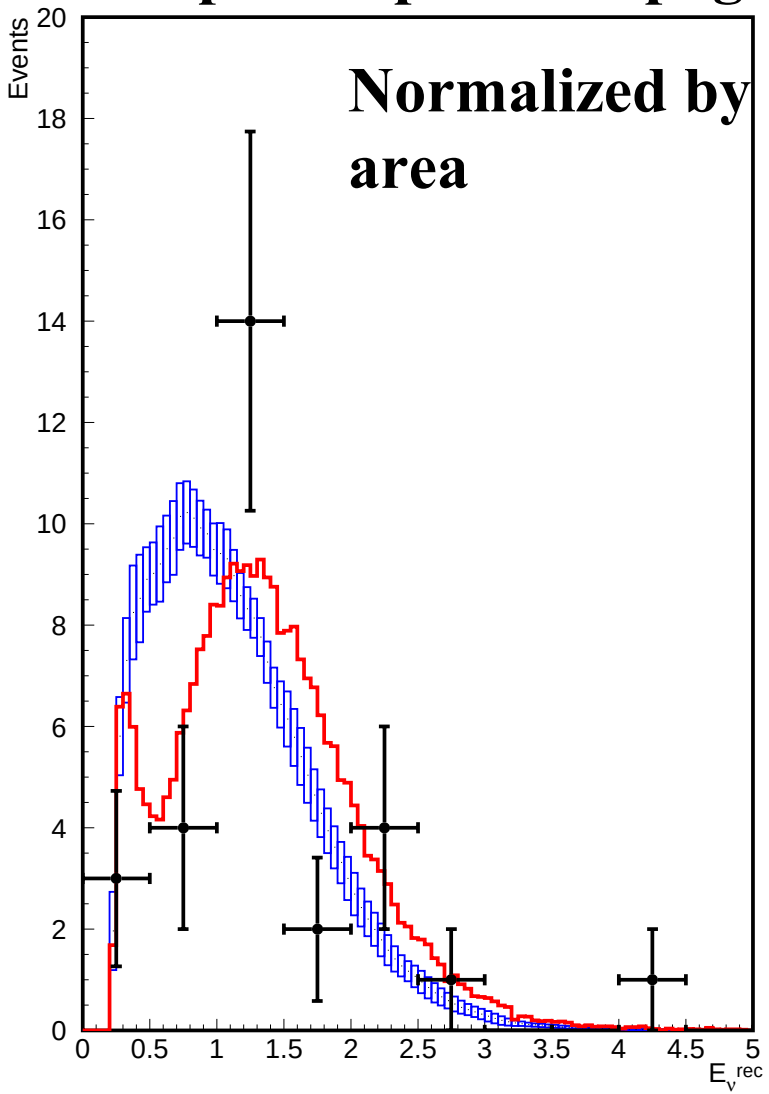
KS test prob: 79%

# of FCFV
fit : 54.2
obs. : 56

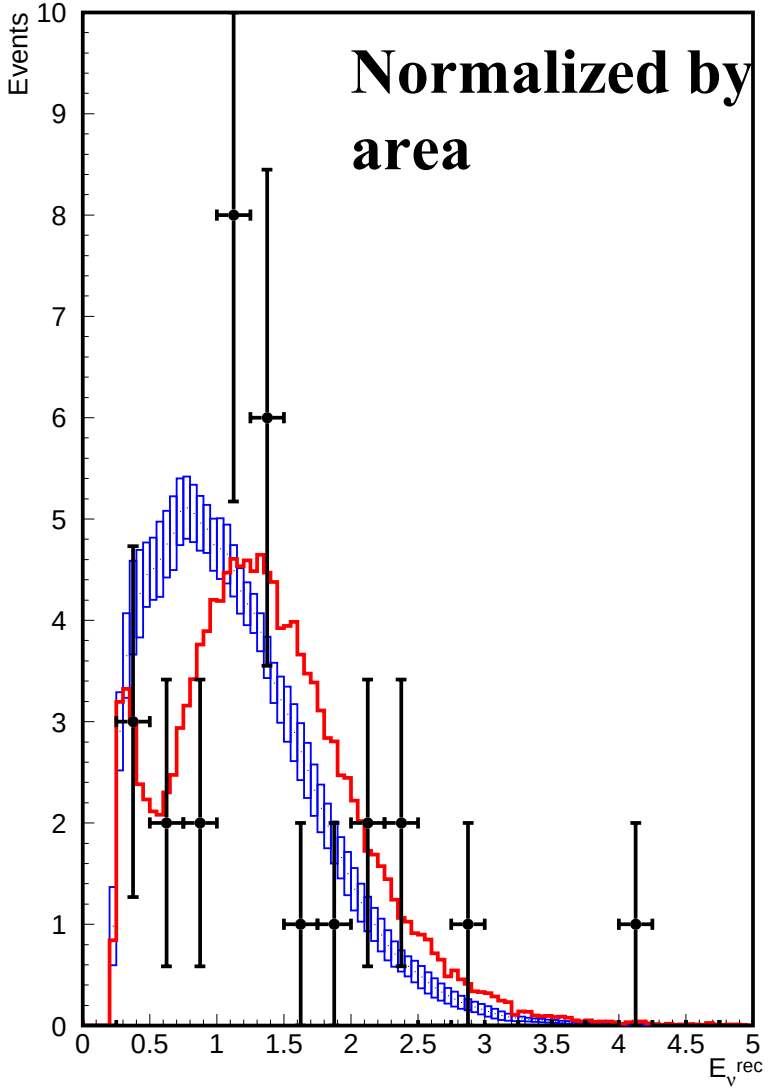
Very good agreement.

Best fit 1Rμ E_ν^{rec} spectrum

Same plot as previous page

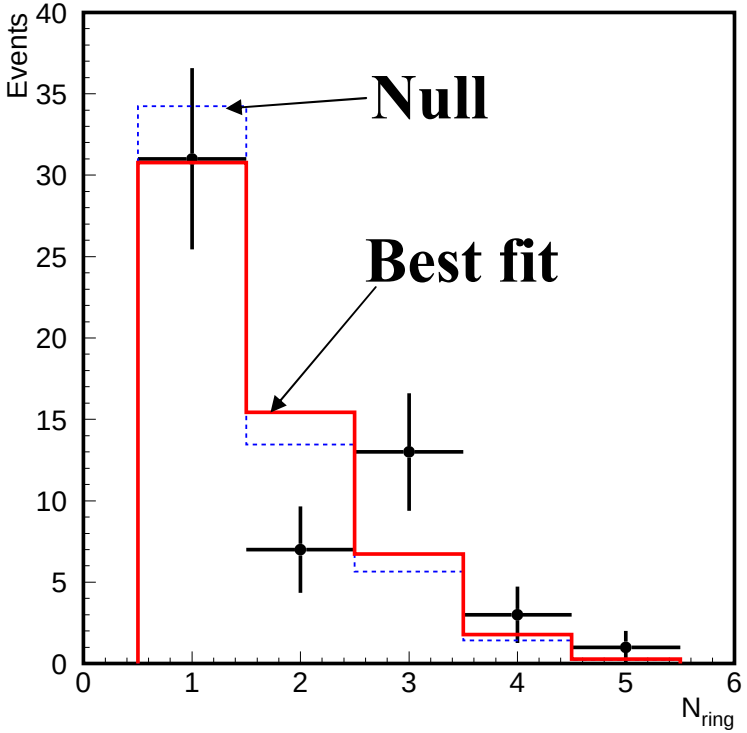


Different binning for data

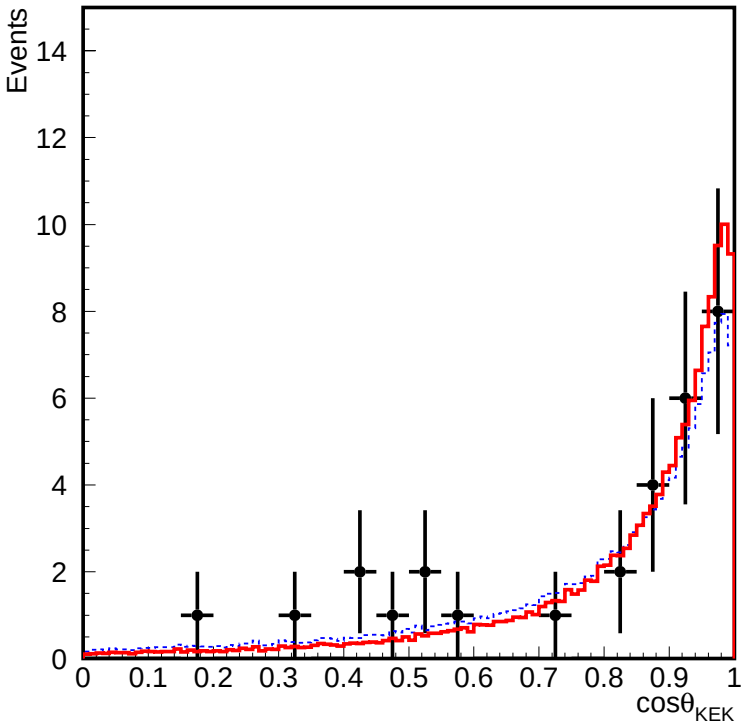


Supplementary plots

of rings for FCFV events



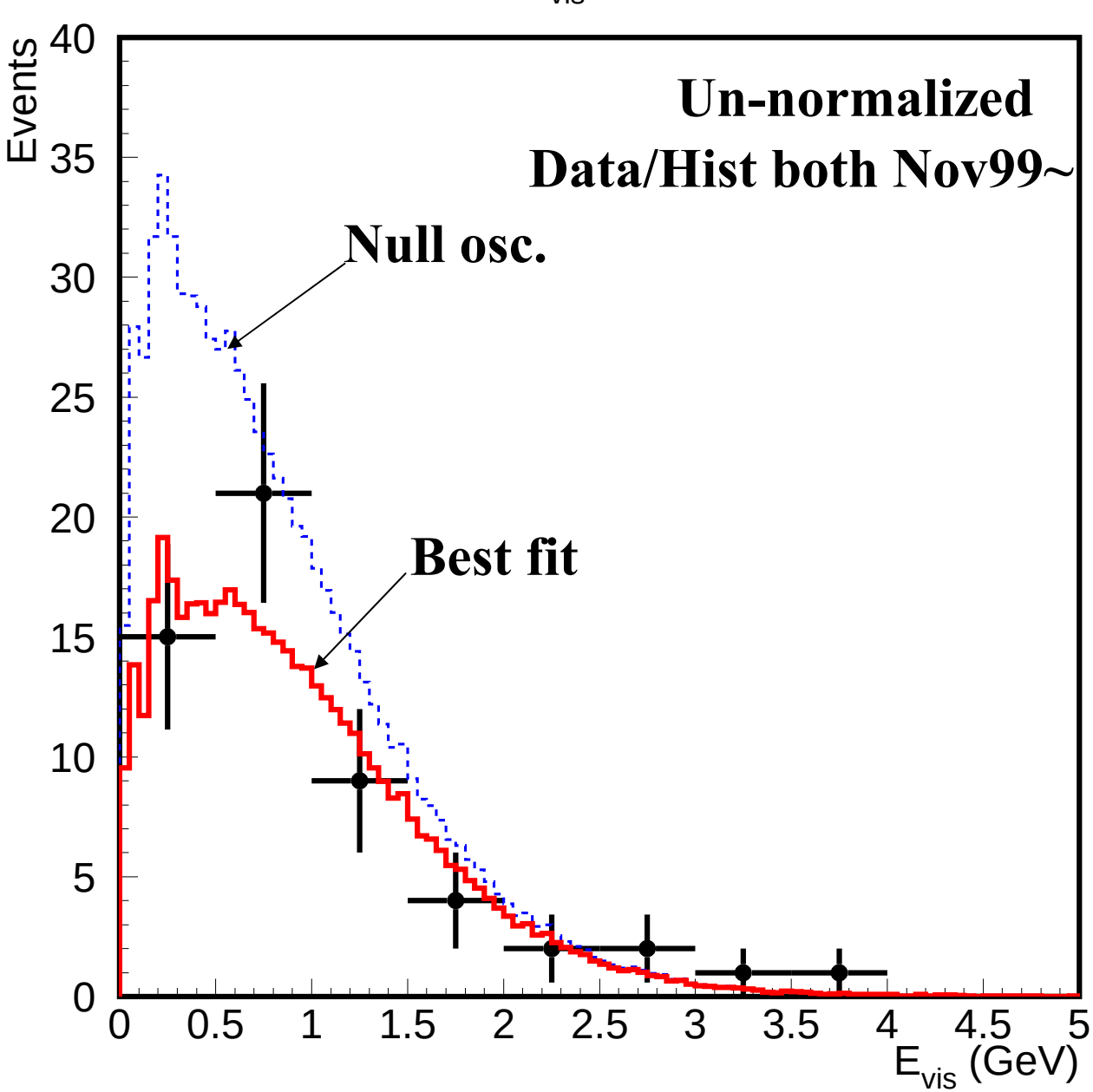
$\cos\theta_{KEK}$ for 1Rm events



Both only Nov99~ data plotted and normalized by area

Red solid: best fit, Blue dashed: null oscillation

Evis distrubtion for FCFV evts



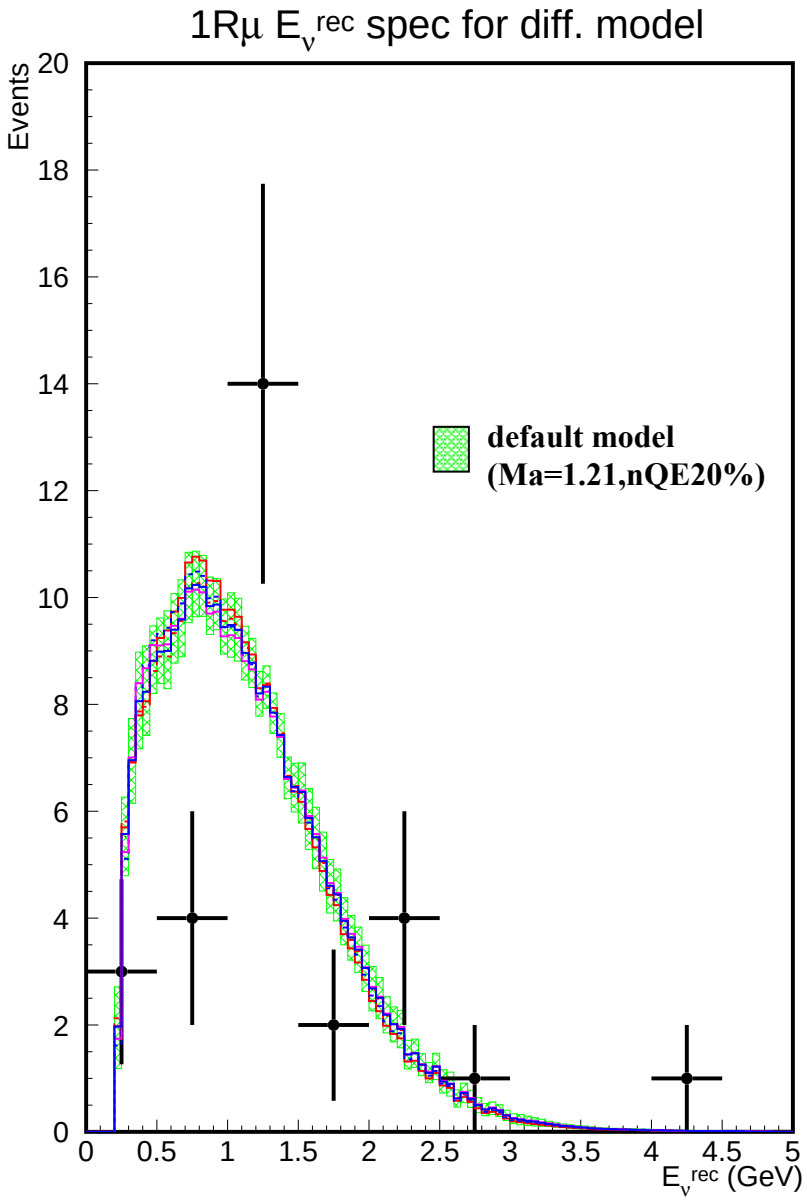
Null probabilities

delta log likelihood method

	dln(L)	Probability
Norm only	**(3.11)**	**1.27%(2side)**
Shape only	**(1.85)**	**15.7%**
N+S	**(4.96)**	**0.7%**

dln(L) calculated from min. in physical region

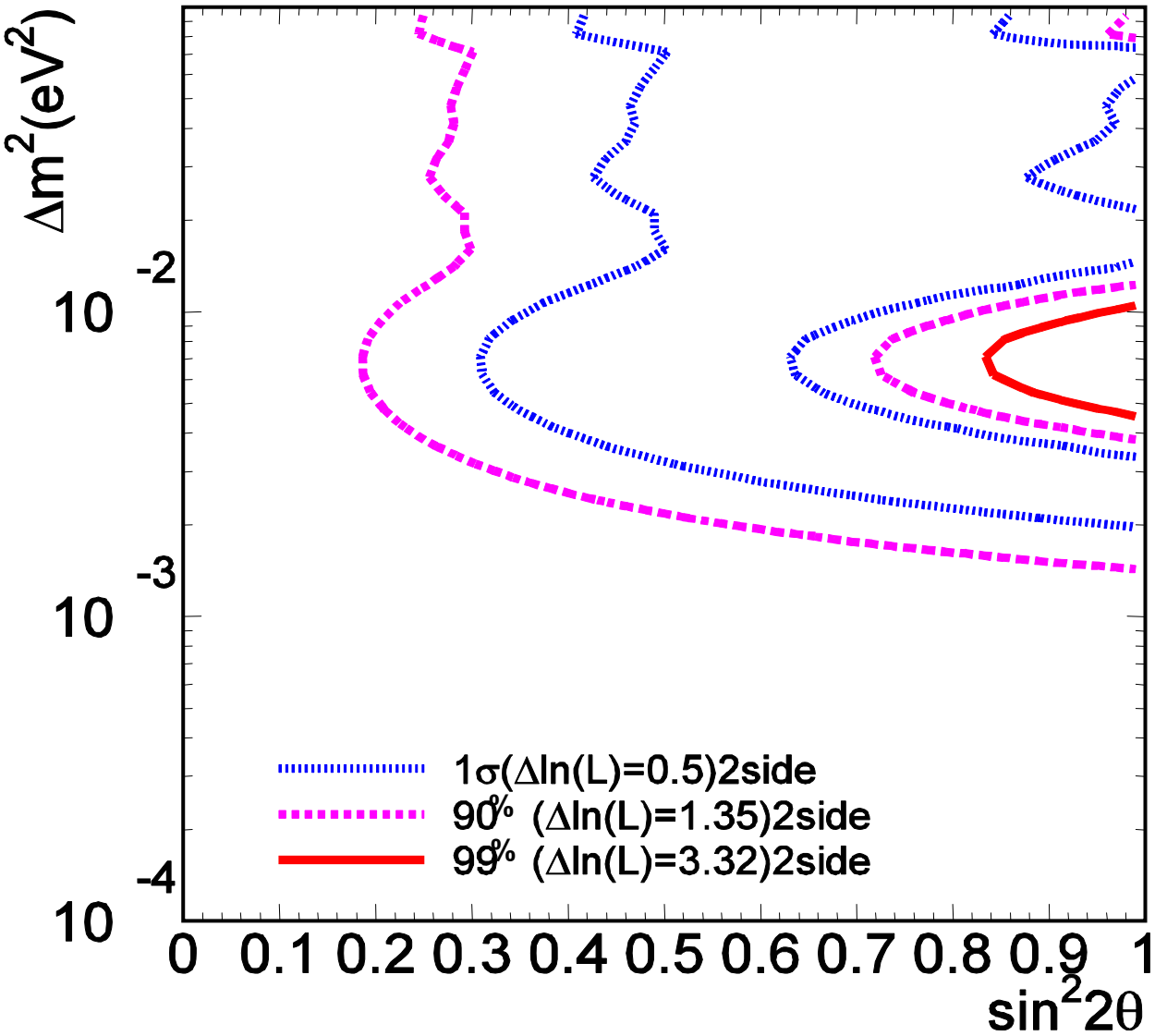
Spectrum difference for various models



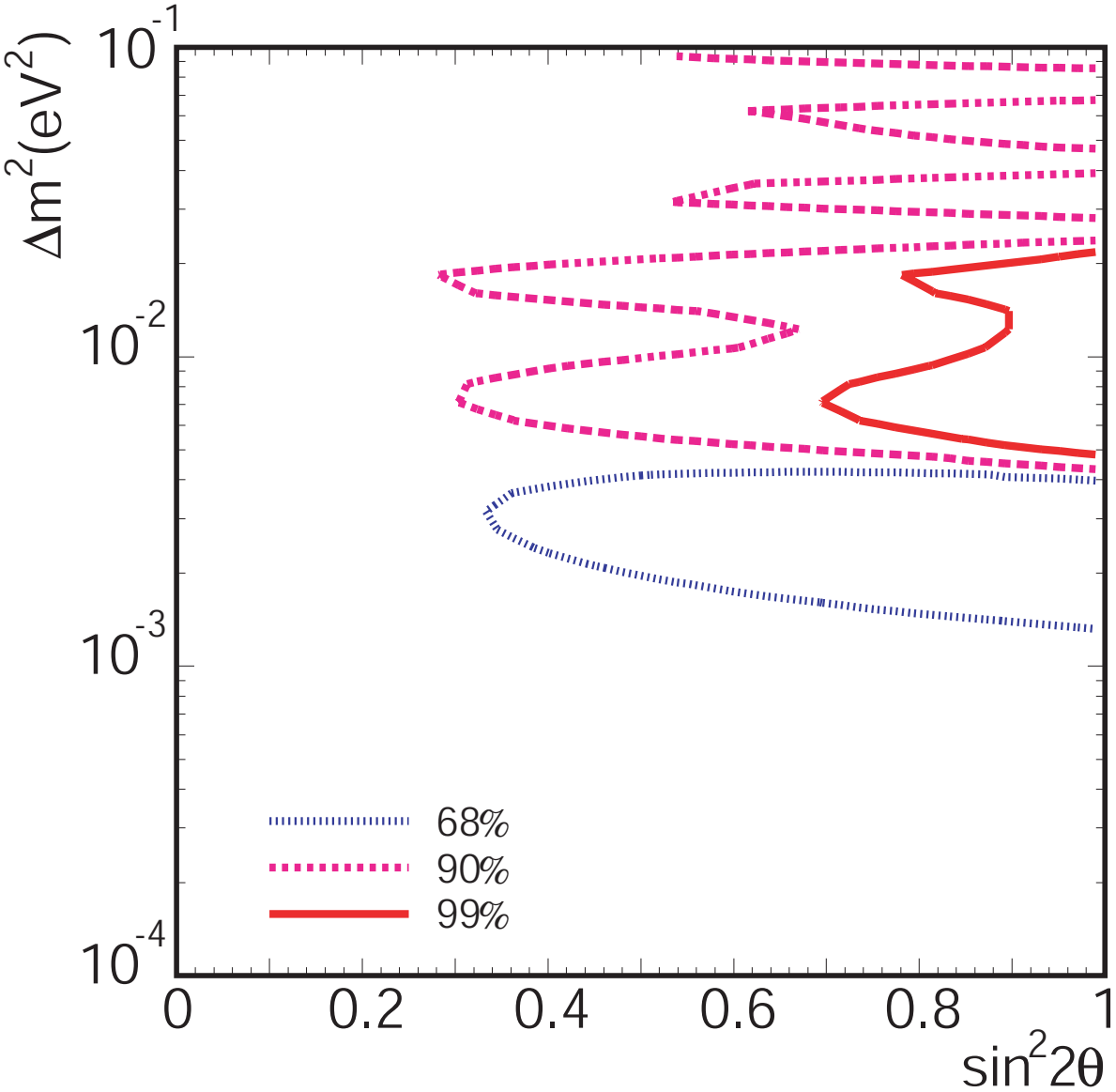
- **Difference is small comp. to statistics**
- **Total # has very small dep. due to cancellation**

Results almost independent of models

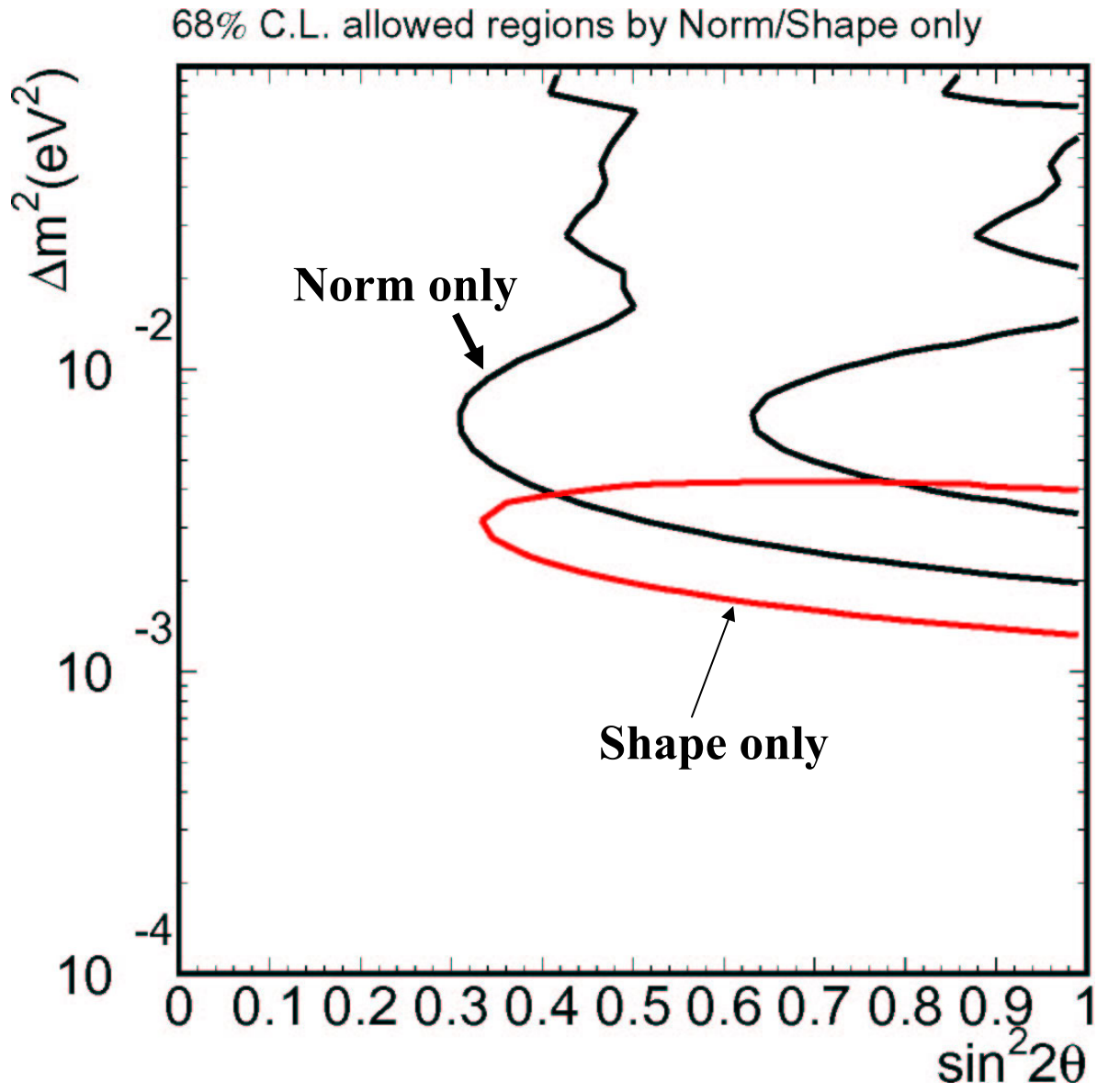
Allowed regions (normalization only)



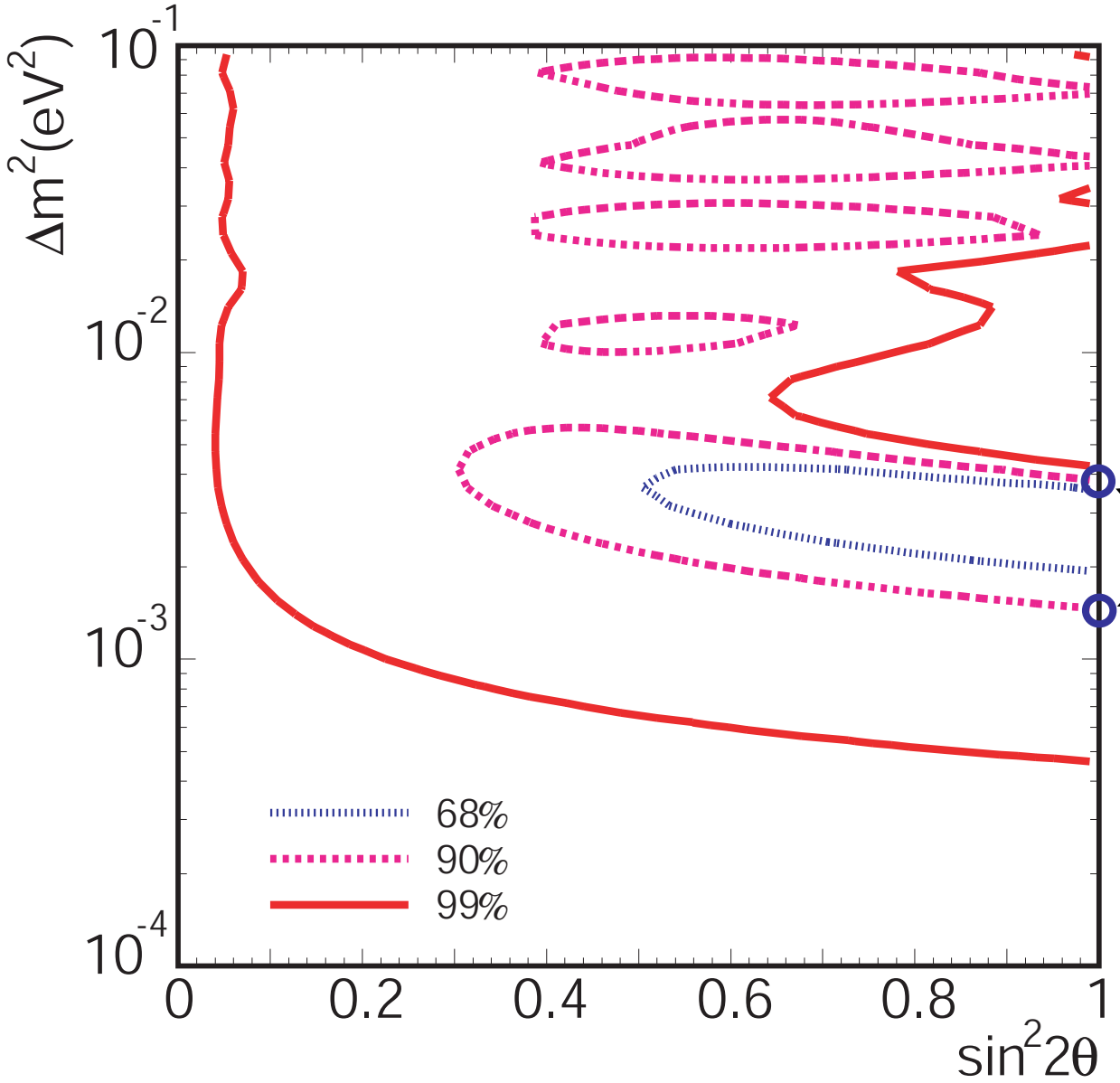
Allowed region (shape only)



Consistency btw shape & norm

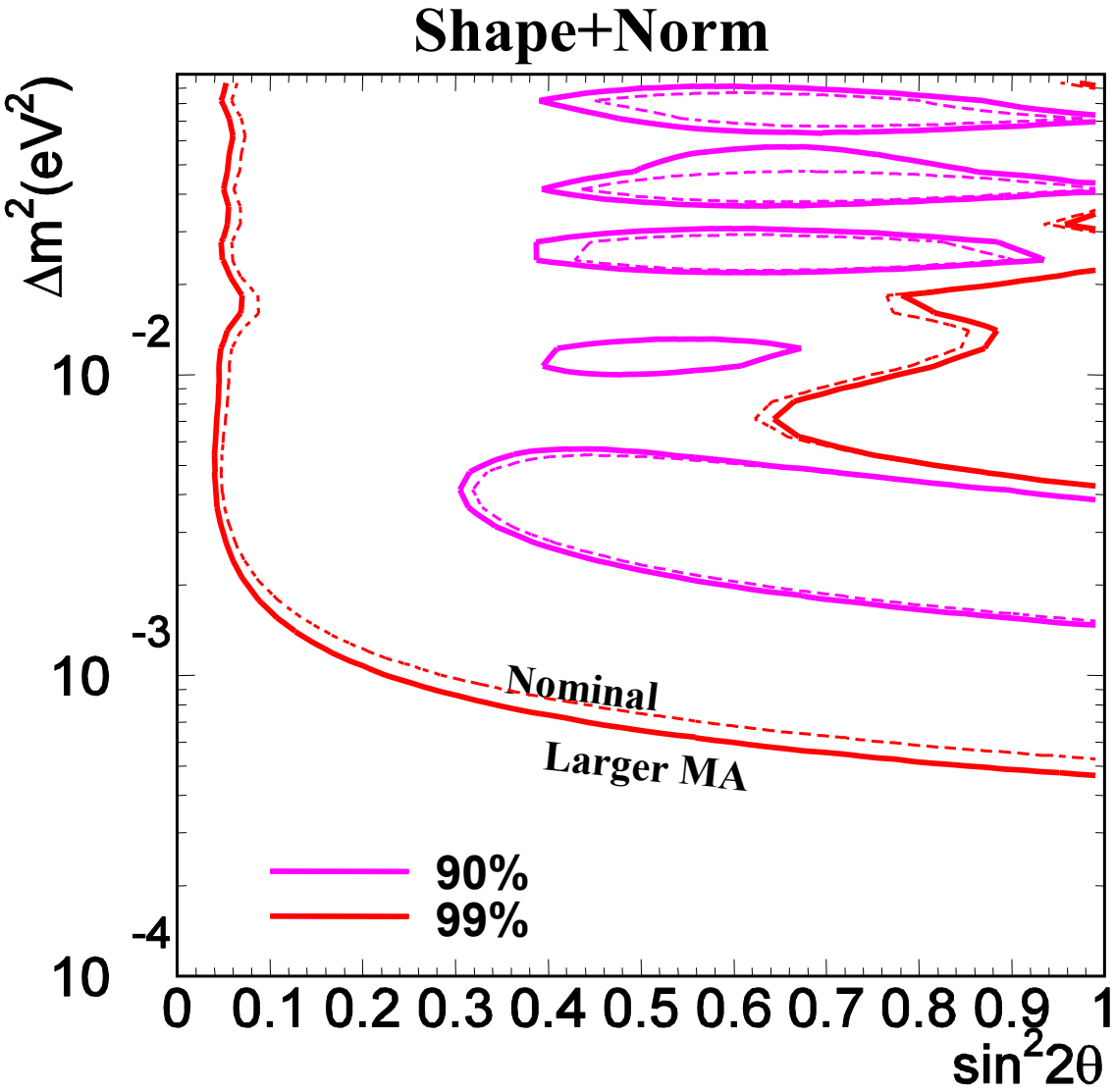


Allowed region (Shape+Norm)



Δm^2= 1.5~3.9x10^{-3}eV^2 @ $\sin^2 2\theta$=1 @90%CL

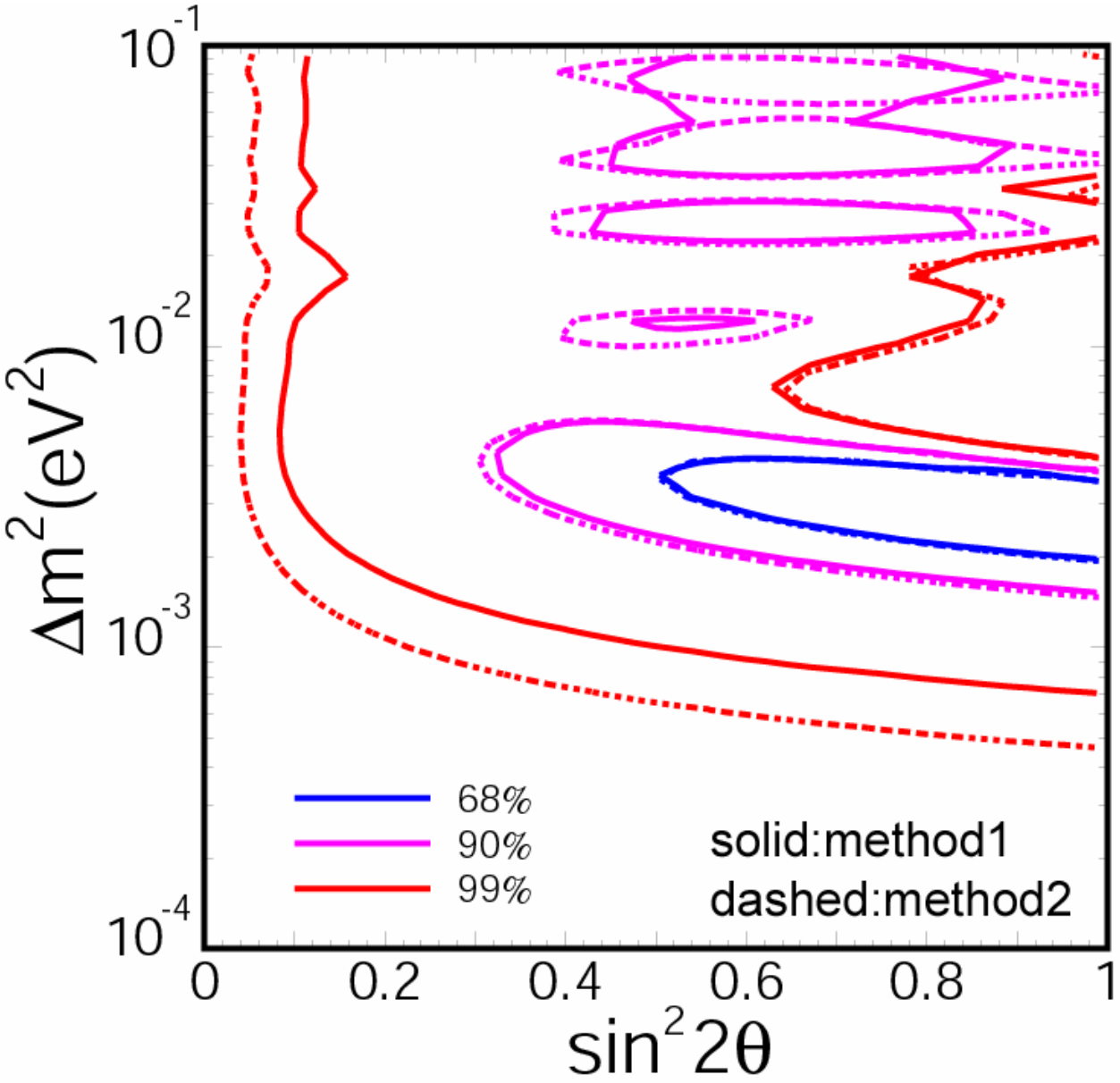
Comparison w/ diff. models



MA=1.21 nQE20%
vs
Nominal model

Small difference.

Comparison btw. diff. methods

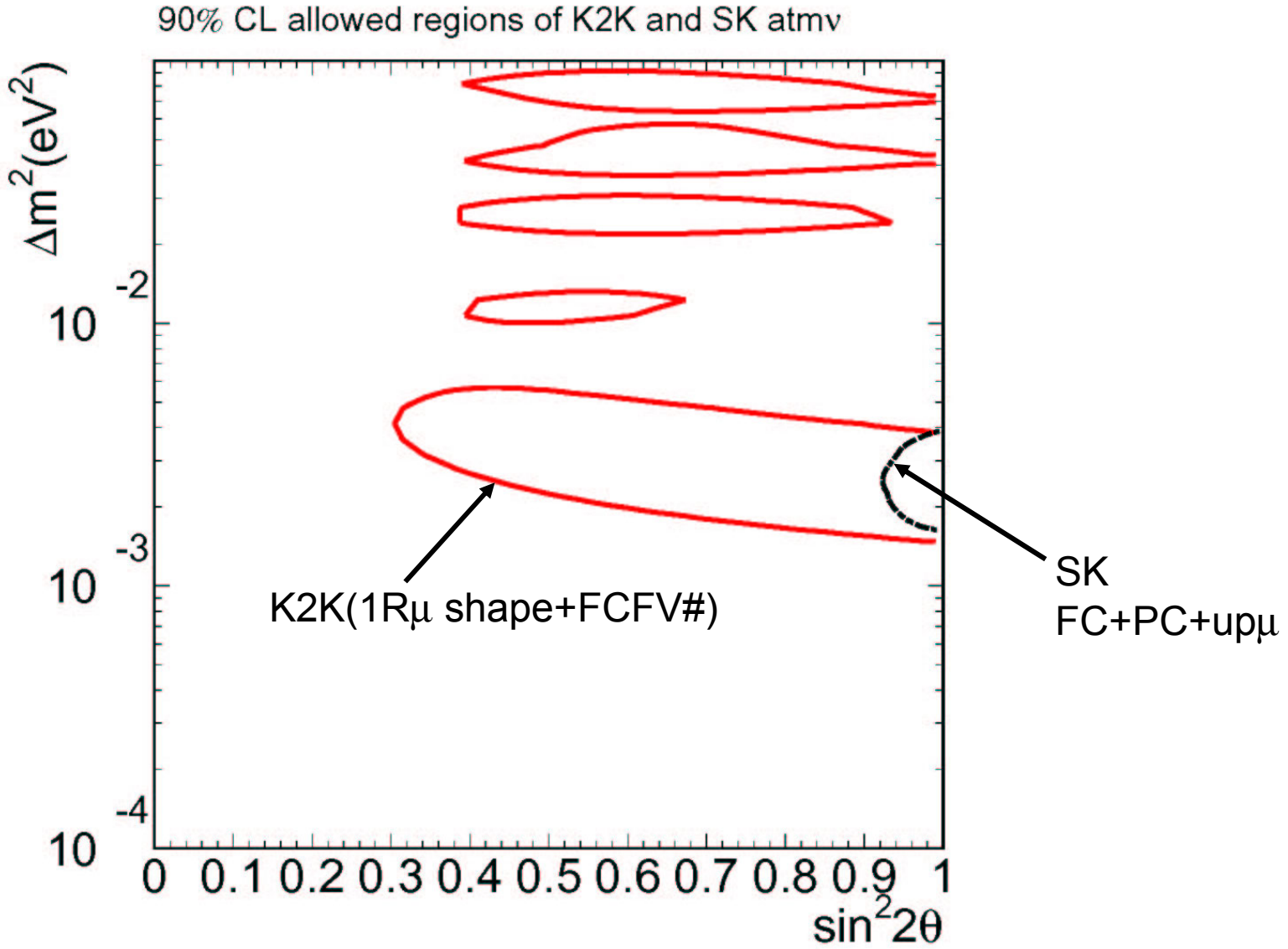


Method1
wgt'ed average of -*lnL* for syst. params.

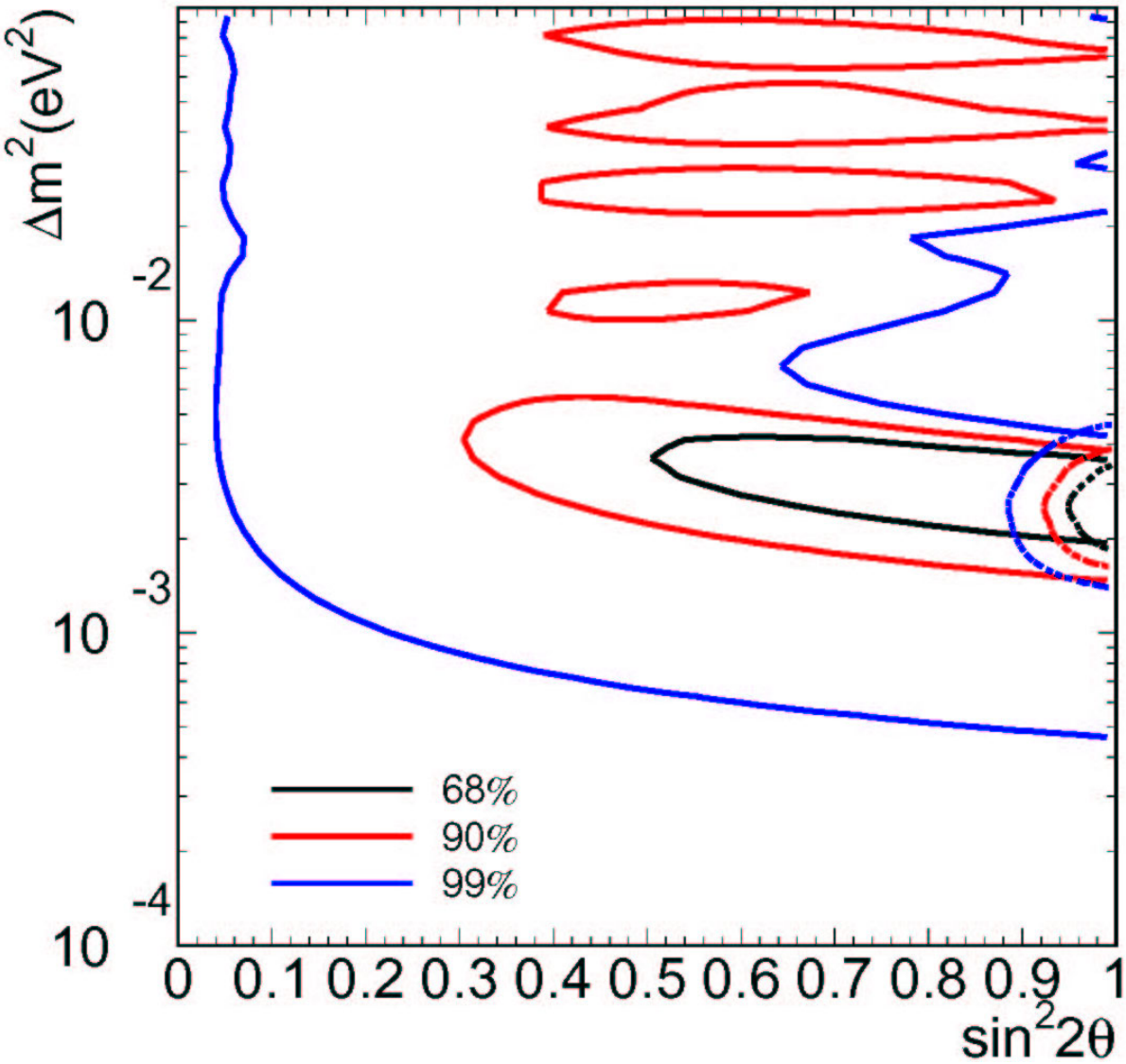
Method2
minimize –*lnL* for syst. params.

Reasonable agreement

Comparison with SK atm ν observation

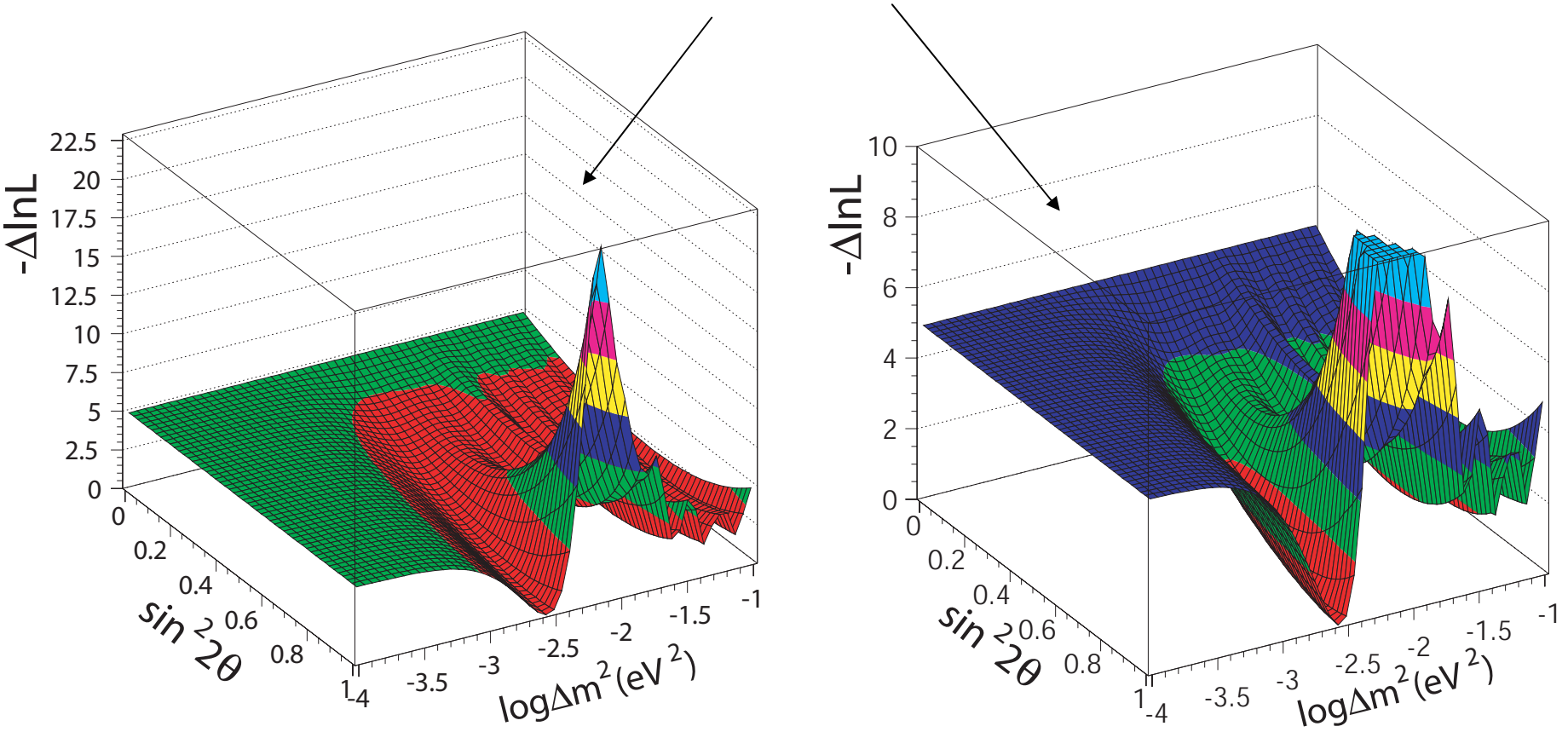


Comparison with SK atm ν observation

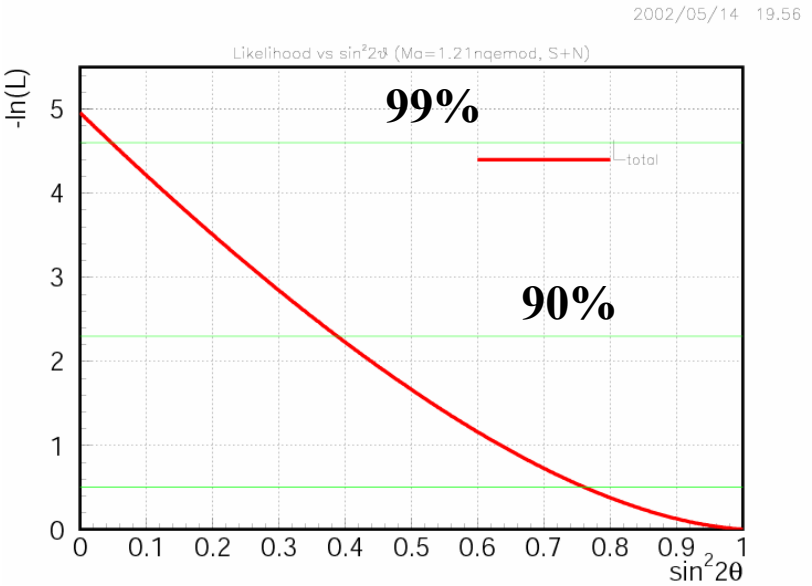


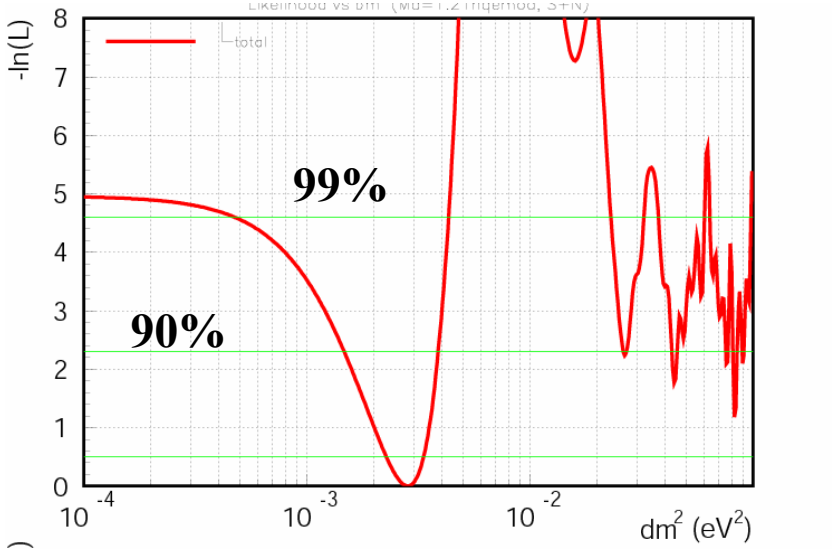
3d plots of ΔlnL for shape+norm

Just Z range is different

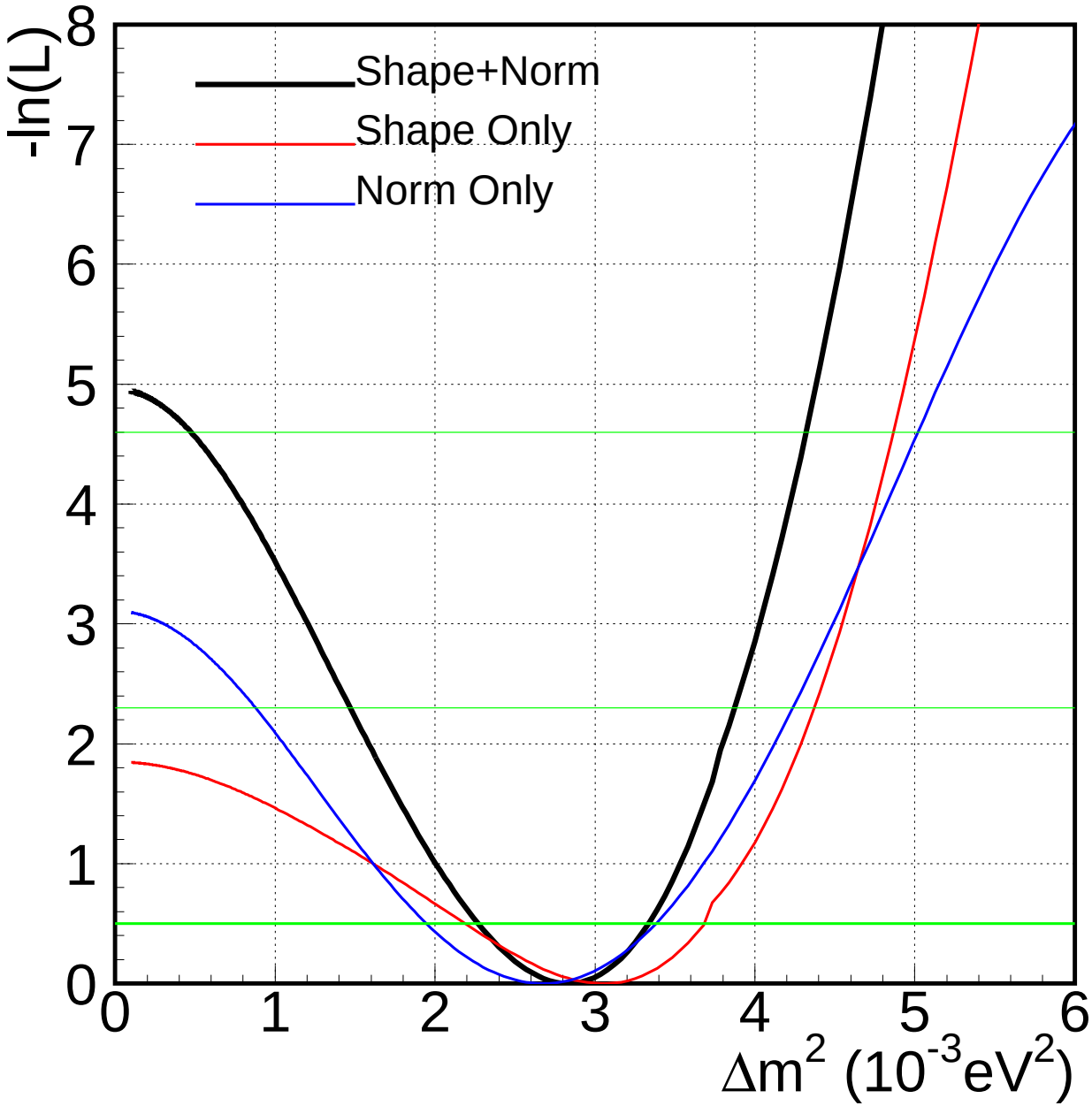


Parameter scan





Shape & Norm both say same Δm^2



Summary

- Max likelihood w/
  - 1Rμ Enrec shape (Nov99~)
  - # of FCFV events (Jun99~)
- Spectrum measurement by FD first applied.
- Best fit
  - $\sin^2 2\theta = 1.03$, $\Delta m^2 = 2.8 \times 10^{-3} eV^2$
  - Fit reproduces observed shape and # of FCFV events very well
- Null prob by delta likelihood
  - **less than 1%**
  - Very small model dependence
- Allowed regions are drawn.
  - $\Delta m^2 = 1.5 \sim 3.9 \times 10^{-3} eV^2$ on $\sin^2 2\theta = 1$ @ 90%CL
  - Shape and Number independently gives consistent parameter region
- Large contribution of errors
  - Far/Near → HARP
  - Overall normalization error of Nsk for Nov99~
  - SK energy scale