Celebrate Seesaw

Seesaw Realization of Bi-Large Mixing and Leptogenesis

Morimitsu TANIMOTO (NiigataUniversity)

Collaborated with M. Honda, S. Kaneko, M. Bando, M. Obara

Fujiwara Seminar, 22 February 2004

Neutrino mass and Seesaw Mechanism

1 Texture Zeros in Neutrino Mass Matrix

2 Seesaw Enhancement of Bi-Large Mixing

3 Thermal Leptogenesis

4 SO(10) Model and Discussions

1 Texture Zeros in Neutrino Mass

Neutrino Experiments suggest Bi-Large Mixings

$$U \approx \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

How to get
Bi-Large Mixing

Yanagida, Altarelli, Raby, Pati, Buchmuller,Bando

Texture Zeros

“ Texture zeros”

Long History in quark mass matrices

$$\tan\theta_C = \sqrt{\frac{m_d}{m_s}}$$

<u>**An empirical relation**</u>

R.J. Oakes, PLB29 (1969) 683

$$M_u = \begin{pmatrix} 0 & A_u \\ A_u & B_u \end{pmatrix}, \quad M_d = \begin{pmatrix} 0 & A_d e^{i\gamma} \\ A_d e^{i\gamma} & B_d \end{pmatrix}$$

Weinberg, Wilzeck, Fritzsch, Ramond, Ross …..

Textures of Two Zeros in Neutrino

Frampton, Glashow, Marfatia, PLB536(2002)79
Xing, …….

7 textures among ${}_6C_2=15$ textures are consistent with experiments

$$A_1:\begin{pmatrix}0&0&\times\\0&\times&\times\\\times&\times&\times\end{pmatrix}\quad A_2:\begin{pmatrix}0&\times&0\\\times&\times&\times\\0&\times&\times\end{pmatrix}$$

Hierarchical neutrino masses

$$B_1:\begin{pmatrix}\times&\times&0\\\times&0&\times\\0&\times&\times\end{pmatrix}\quad B_2:\begin{pmatrix}\times&0&\times\\0&\times&\times\\\times&\times&0\end{pmatrix}\quad B_3:\begin{pmatrix}\times&0&\times\\0&0&\times\\\times&\times&\times\end{pmatrix}\quad B_4:\begin{pmatrix}\times&\times&0\\\times&\times&\times\\0&\times&0\end{pmatrix}\quad C:\begin{pmatrix}\times&\times&\times\\\times&0&\times\\\times&\times&0\end{pmatrix}$$

x ：non-zero entry

Charged Lepton Mass Matrix is diagonal.

9 – 4 (2 zeros)= 5 parameters

$$\mathbf{A_1}: \begin{pmatrix} 0 & 0 & \epsilon \\ 0 & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \qquad \mathbf{A_2}: \begin{pmatrix} 0 & \epsilon & 0 \\ \epsilon & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

Why does A1, A2 give two large mixings and small Ue3?

After maximal rotation of (2-3) elements

$$\mathbf{A_1}: \begin{pmatrix} 0 & \frac{1}{\sqrt{2}}\epsilon & -\frac{1}{\sqrt{2}}\epsilon \\ \frac{1}{\sqrt{2}}\epsilon & \epsilon' & 0 \\ -\frac{1}{\sqrt{2}}\epsilon & 0 & 2 \end{pmatrix} \qquad \mathbf{A_2}: \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}}\epsilon & \frac{1}{\sqrt{2}}\epsilon \\ -\frac{1}{\sqrt{2}}\epsilon & \epsilon' & 0 \\ \frac{1}{\sqrt{2}}\epsilon & 0 & 2 \end{pmatrix}$$

$$\frac{\Delta m^2_{\mathrm{sun}}}{\Delta m^2_{\mathrm{atm}}} \simeq \epsilon^2 \qquad \epsilon = 0.1 \sim 0.2 \qquad U_{\mathrm{e3}} = O(\epsilon)$$

$\tan^2\theta_{\mathrm{sun}} \simeq O(1)$ **depends on** ϵ/ϵ'

Since we have only 5 parameters, we get testable relation

$$(M_\nu)_{\alpha\beta} = \sum_{i=1}^{3} U_{\alpha i} U_{\beta i} \lambda_i$$

$$\lambda_1 = m_1 e^{2i\rho}, \ \ \lambda_2 = m_2 e^{2i\sigma}, \ \ \lambda_3 = m_3$$

A2 type $\qquad (M_\nu)_{ee} = 0, \quad (M_\nu)_{e\tau} = 0$

$$\frac{m_1}{m_2} = \left| \frac{s_{13}}{c_{13}^2} \left(\frac{s_{12} c_{23}}{c_{12} s_{23}} + \mathrm{s_{13} e^{-i\delta}} \right) \right| , \quad \frac{m_2}{m_3} = \left| \frac{s_{13}}{c_{13}^2} \left(\frac{c_{12} c_{23}}{s_{12} s_{23}} - \mathrm{s_{13} e^{-i\delta}} \right) \right|$$

$$|\mathbf{U_{e3}}| = \frac{1}{2} \tan 2\theta_{12} \tan \theta_{23} \sqrt{R_\nu \cos 2\theta_{12}}, \quad \mathbf{R}_\nu = \Delta \mathbf{m}^2_{\mathbf{sun}} / \Delta \mathbf{m}^2_{\mathbf{atm}}$$

2 Seesaw Enhancement of Bi-LargeMixing

Seesaw Decomposition of Texture Zeros

$$M_\nu = m_D M_R^{-1} m_D^T$$

Suppose: Zeros come from zeros in Dirac mass matrix and Right-handed Majorana mass matrix

M_R is fixed $\Rightarrow$ m_D is given (many combinations)

Remark: $M_\nu = \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}$ cannot be reproduced

if $M_R : \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix} \quad \begin{pmatrix} 0 & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix}$

Seesaw Enhancement

Even if small flavor mixing in m_D and M_R,

$m_D M_R$^-1 m_D^T could has large mixings

Smirnov, PRD48(1993)3264; Tanimoto, PLB345(1995)477

In the framework of Two Zero Texture,
Seesaw Enhancement Textures are easily obtained.

Typical Example is

$$\mathbf{M_R} = \mathbf{M_3} \begin{pmatrix} 0 & \lambda^{\frac{m+n}{2}} & 0 \\ \lambda^{\frac{m+n}{2}} & -\lambda^{n} & 0 \\ 0 & 0 & 1 \end{pmatrix}_{\mathbf{RR}} \qquad \mathbf{m_D} = \mathbf{m_{D0}} \begin{pmatrix} 0 & \lambda^{\frac{n+2}{2}} & 0 \\ \lambda^{\frac{m}{2}} & 0 & 0 \\ 0 & \lambda^{\frac{n}{2}} & 1 \end{pmatrix}_{\mathbf{LR}}$$

$$|\mathbf{M_1}| = \lambda^{\mathbf{m}}\mathbf{M_3}, \quad |\mathbf{M_2}| = \lambda^{\mathbf{n}}\mathbf{M_3}; \qquad \mathbf{M}_\nu \sim \begin{pmatrix} 0 & \lambda & 0 \\ \lambda & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

A2 case

Clearly find

$\mu \to e\gamma$ **suppression and 1 CP violating phase**

$$|M_1| = \lambda^m M_3, \quad |M_2| = \lambda^n M_3; \qquad m > n > 0$$

$$M_R \simeq M_3 \begin{pmatrix} 0 & 0 & \lambda^{\frac{m}{2}} \\ 0 & \lambda^n & 0 \\ \lambda^{\frac{m}{2}} & 0 & -1 \end{pmatrix}_{a3} \qquad m_D = m_{D0} \begin{pmatrix} 0 & 0 & \lambda \\ \lambda^{\frac{m}{2}} & 0 & 0 \\ 0 & \lambda^{\frac{n}{2}} & 1 \end{pmatrix}_{a3}$$

$$M_R \simeq M_3 \begin{pmatrix} -\lambda^n & \lambda^{\frac{m+n}{2}} & 0 \\ \lambda^{\frac{m+n}{2}} & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}_{b1} \qquad m_D = m_{D0} \begin{pmatrix} \lambda^{\frac{n}{2}+1} & 0 & 0 \\ 0 & \lambda^{\frac{m}{2}} & 0 \\ \lambda^{\frac{n}{2}} & 0 & 1 \end{pmatrix}_{b1}$$

$$M_R \simeq M_3 \begin{pmatrix} 0 & \lambda^{\frac{m+n}{2}} & -\lambda^{\frac{n}{2}} \\ \lambda^{\frac{m+n}{2}} & 0 & 0 \\ -\lambda^{\frac{n}{2}} & 0 & 1 \end{pmatrix}_{b2} \qquad m_D = m_{D0} \begin{pmatrix} \lambda^{\frac{n}{2}+1} & 0 & 0 \\ 0 & \lambda^{\frac{m}{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}_{b2}$$

Maximal Number of Zeros in m_D

$$M_R \simeq M_3 \begin{pmatrix} 0 & \lambda^{\frac{m+n}{2}} & 0 \\ \lambda^{\frac{m+n}{2}} & -\lambda^{n} & 0 \\ 0 & 0 & 1 \end{pmatrix}_{b3} \qquad m_D = m_{D0} \begin{pmatrix} 0 & \lambda^{\frac{n}{2}+1} & 0 \\ \lambda^{\frac{m}{2}} & 0 & 0 \\ 0 & \lambda^{\frac{n}{2}} & 1 \end{pmatrix}_{b3}$$

$$M_R \simeq M_3 \begin{pmatrix} 0 & \lambda^{\frac{m+n}{2}} & 0 \\ \lambda^{\frac{m+n}{2}} & 0 & -\lambda^{\frac{n}{2}} \\ 0 & -\lambda^{\frac{n}{2}} & 1 \end{pmatrix}_{b4} \qquad m_D = m_{D0} \begin{pmatrix} 0 & \lambda^{\frac{n}{2}+1} & 0 \\ \lambda^{\frac{m}{2}} & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}_{b4}$$

$$M_R \simeq M_3 \begin{pmatrix} \lambda^{m} & 0 & 0 \\ 0 & 0 & \lambda^{\frac{n}{2}} \\ 0 & \lambda^{\frac{n}{2}} & -1 \end{pmatrix}_{c3} \qquad m_D = m_{D0} \begin{pmatrix} 0 & 0 & \lambda \\ 0 & \lambda^{\frac{n}{2}} & 0 \\ \lambda^{\frac{m}{2}} & 0 & 1 \end{pmatrix}_{c3}$$

There are other 6 sets of M_R and m_D

$\mu \to e\gamma$ Process in SUSY

$$\Gamma(\mu \to \mathrm{e} + \gamma) \simeq \frac{e^2}{256\pi^3 v_2^2} \mathrm{m}_\mu^5 \mathrm{F} \left| (6 + 2\mathrm{a}_0^2) \mathrm{m}_{\mathrm{S0}}^2 (\mathrm{m_D m_D^\dagger})_{21} \ln \frac{M_{\mathrm{GUT}}}{M_R} \right|^2$$

Experimental Bound : $\boxed{Br(\mu \to e\gamma) < 1.2 \times 10^{-11}}$

Bi-Large Mixing gives Large Branching Ratio in general

However , our textures suppress branching ratio because

$$(\mathrm{m_D m_D^\dagger})_{21} = 0, \quad (\mathrm{m_D m_D^\dagger})_{31} (\mathrm{m_D m_D^\dagger})_{23} = 0$$

$$\mathrm{m_D} = \mathrm{m_{D0}} \begin{pmatrix} 0 & \lambda^{\frac{n+2}{2}} & 0 \\ \lambda^{\frac{m}{2}} & 0 & 0 \\ 0 & \lambda^{\frac{n}{2}} & 1 \end{pmatrix}_{\mathrm{LR}}$$

Physics in Left-handed
MR independent

3 Thermal Leptogenesis

Majorana Mass Term $|\Delta L| = 2$

CP Violation

Out of Equilibrium Decay of N_R

Lepton Asymmetry

In See-saw, there are 3 M_R masses (M_1, M_2, M_3), 9 Real Dirac mass components and 6 CP phases. Total: 18 parameters.

9 parameters are integrated out in M_ν

It is difficult to discuss the link between the lepton asymmetry and low energy CP violation J_{CP} in general.

However, Zeros in the mass matrices reduce the number of parameters.

$$\epsilon_1 \equiv \frac{\Gamma(N_1 \to \ell H) - \Gamma(N_1 \to \overline{\ell H})}{\Gamma(N_1 \to \ell H) + \Gamma(N_1 \to \overline{\ell H})} \neq 0$$ one-loop

$$\epsilon_1 = \frac{1}{8\pi v_2^2}\frac{1}{(m_D^\dagger m_D)_{11}}\sum_{\mathrm{i}\neq 1} \mathrm{Im}[(\mathrm{m_D^\dagger m_D})^2_{1\mathrm{i}}][\mathrm{f(x_i) + g(x_i)}]$$

where $\mathrm{x_i = M_i^2/M_1^2}$ f(x) and g(x) are loop-functions

Physics in Right-handed sector: M_E independent

$$\mathrm{Y_L} \equiv \frac{n_L - n_{\overline{L}}}{s} = \kappa \frac{\epsilon_1}{g*}$$

κ is dilution factor

g* is number of relativistic degrees of freedom

Through (B+L)-violating sphaleron processes

Fukugita-Yanagida

$$\mathrm{Y_B} = -\frac{8}{15}\mathrm{Y_L}$$

in the MSSM

$$\mathbf{m_D} = \mathbf{m_{D0}} \begin{pmatrix} 0 & \lambda^{\frac{n}{2}+1} & 0 \\ \lambda^{\frac{m}{2}} & 0 & 0 \\ 0 & \lambda^{\frac{n}{2}} e^{i\rho} & 1 \end{pmatrix}_{\mathbf{b3}} \qquad \mathrm{M_R} \simeq \mathrm{M_3} \begin{pmatrix} 0 & \lambda^{\frac{m+n}{2}} & 0 \\ \lambda^{\frac{m+n}{2}} & -\lambda^{n} & 0 \\ 0 & 0 & 1 \end{pmatrix}_{\mathbf{b3}}$$

$$\epsilon_1 \simeq -\frac{3m_{D0}^2}{8\pi v_2^2}\,\lambda^{\mathbf{m}}\ \sin 2\rho \simeq -8.8\times 10^{-17}\left(\frac{M_1}{1\mathrm{GeV}}\right)\sin 2\rho$$

$$\mathrm{J_{CP}} \simeq \frac{1}{64}\,\lambda^2\ \frac{\Delta m^2_{\mathrm{atm}}}{\Delta m^2_{\mathrm{sun}}}\sin 2\rho \sim 0.01$$

WMAP $\eta_{\mathrm{B}} = 6.5^{+0.4}_{-0.3}\times 10^{-10}(1\sigma) \Longrightarrow \mathrm{M_1}\sin 2\rho \simeq 6\times 10^{10}\,\mathrm{GeV}$

Is our textures consistent with GUT model?

$$\mathbf{m_D} = \mathbf{m_{D0}} \begin{pmatrix} 0 & \lambda^{\frac{n}{2}+1} & 0 \\ \lambda^{\frac{m}{2}} & \lambda^{x} & \lambda^{y} \\ 0 & \lambda^{\frac{n}{2}} & 1 \end{pmatrix}_{\mathbf{b3'}} \qquad \mathbf{M_R} \simeq \mathbf{M_3} \begin{pmatrix} 0 & \lambda^{\frac{m+n}{2}} & 0 \\ \lambda^{\frac{m+n}{2}} & -\lambda^{n} & 0 \\ 0 & 0 & 1 \end{pmatrix}_{\mathbf{b3}}$$

As far as x>n/2 and y>0, A2 type texture is reproduced.

Putting y = n/2 , m= n+2, m_D becomes symmetric matrix.

Let us discuss

SO(10) model with texture Zeros

4 SO(10) Model and Discussions

M. Bando and M.Obara, Prog. Theor. Phys. 109 (2003) 995

M. Bando, S. Kaneko, M. Obara, M. Tanimoto, Phys.Lett. B580 (2004) 229

Down-type sector

$$M_D, M_l; \begin{pmatrix} 0 & 10 & 0 \\ 10 & 126 & 10 \\ 0 & 10 & 10 \end{pmatrix}$$

→ **Georgi-Jarlskog relation**

$$M_D = \begin{pmatrix} 0 & a_d & 0 \\ a_d & b_d & c_d \\ 0 & c_d & d_d \end{pmatrix}$$

$$M_l = \begin{pmatrix} 0 & a_d & 0 \\ a_d & -3b_d & c_d \\ 0 & c_d & d_d \end{pmatrix}$$

Ratio of Yukawa couplings

Higgs	Quark		Lepton
10	1	:	1
126	1	:	−3

Up-type sector

$$M_U, M_{\nu_D}; \begin{pmatrix} 0 & & 0 \\ & ? & \\ 0 & & \end{pmatrix}$$

16 types of textures

$$M_U \simeq \begin{pmatrix} 0 & \sqrt{m_u m_c} & 0 \\ \sqrt{m_u m_c} & m_c & \sqrt{m_u m_t} \\ 0 & \sqrt{m_u m_t} & m_t \end{pmatrix} \equiv m_t \begin{pmatrix} 0 & a_u & 0 \\ a_u & b_u & c_u \\ 0 & c_u & 1 \end{pmatrix}$$

$$(m_u \ll m_c \ll m_t)$$

$$M_{\nu_D} \simeq m_t \begin{pmatrix} 0 & *a_u & 0 \\ *a_u & *b_u & *c_u \\ 0 & *c_u & * \end{pmatrix} \equiv m_t \begin{pmatrix} 0 & a & 0 \\ a & b & c \\ 0 & c & 1 \end{pmatrix}$$

$*$:1 or -3

16 types of textures

Class	Type 1	Type 2	Type 3	Type 4
S	$\begin{pmatrix} 0 & 126 & 0 \\ 126 & 10 & 10 \\ 0 & 10 & 126 \end{pmatrix}$	$\begin{pmatrix} 0 & 126 & 0 \\ 126 & 10 & 10 \\ 0 & 10 & 10 \end{pmatrix}$		
A	$\begin{pmatrix} 0 & 126 & 0 \\ 126 & 126 & 126 \\ 0 & 126 & 126 \end{pmatrix}$	$\begin{pmatrix} 0 & 126 & 0 \\ 126 & 126 & 126 \\ 0 & 126 & 10 \end{pmatrix}$	$\begin{pmatrix} 0 & 10 & 0 \\ 10 & 10 & 10 \\ 0 & 10 & 126 \end{pmatrix}$	$\begin{pmatrix} 0 & 10 & 0 \\ 10 & 10 & 10 \\ 0 & 10 & 10 \end{pmatrix}$
B	$\begin{pmatrix} 0 & 10 & 0 \\ 10 & 126 & 126 \\ 0 & 126 & 126 \end{pmatrix}$	$\begin{pmatrix} 0 & 10 & 0 \\ 10 & 126 & 126 \\ 0 & 126 & 10 \end{pmatrix}$		
C	$\begin{pmatrix} 0 & 126 & 0 \\ 126 & 10 & 126 \\ 0 & 126 & 126 \end{pmatrix}$	$\begin{pmatrix} 0 & 10 & 0 \\ 10 & 10 & 126 \\ 0 & 126 & 126 \end{pmatrix}$	$\begin{pmatrix} 0 & 10 & 0 \\ 10 & 10 & 126 \\ 0 & 126 & 10 \end{pmatrix}$	$\begin{pmatrix} 0 & 126 & 0 \\ 126 & 10 & 126 \\ 0 & 126 & 10 \end{pmatrix}$
F	$\begin{pmatrix} 0 & 126 & 0 \\ 126 & 126 & 10 \\ 0 & 10 & 126 \end{pmatrix}$	$\begin{pmatrix} 0 & 10 & 0 \\ 10 & 126 & 10 \\ 0 & 10 & 126 \end{pmatrix}$	$\begin{pmatrix} 0 & 10 & 0 \\ 10 & 126 & 10 \\ 0 & 10 & 10 \end{pmatrix}$	$\begin{pmatrix} 0 & 126 & 0 \\ 126 & 126 & 10 \\ 0 & 10 & 10 \end{pmatrix}$

Texture of M_R

$$M_R = \begin{pmatrix} 0 & r & 0 \\ r & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_R,$$

$$M_R = \begin{pmatrix} 0 & r & 0 \\ r & s & 0 \\ 0 & 0 & 1 \end{pmatrix} m_R, \qquad M_R = \begin{pmatrix} 0 & r & 0 \\ r & 0 & t \\ 0 & t & 1 \end{pmatrix} m_R,$$

$$M_R = \begin{pmatrix} 0 & r & 0 \\ r & s & t \\ 0 & t & 1 \end{pmatrix} m_R.$$

These lead to A_2 type texture of M_ν

Let us show the simplest case for M_R

$$M_R = \begin{pmatrix} 0 & r & 0 \\ r & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_R$$

See-saw Mechanism gives

$$\Rightarrow \quad M_\nu = \begin{pmatrix} 0 & \frac{a^2}{r} & 0 \\ \frac{a^2}{r} & 2\frac{ab}{r} + c^2 & c(\frac{a}{r}+1) \\ 0 & c(\frac{a}{r}+1) & d^2 \end{pmatrix} \frac{m_t^2}{m_R}$$

$$M_\nu = \begin{pmatrix} 0 & \frac{a^2}{r} & 0 \\ \frac{a^2}{r} & 2\frac{ab}{r} + c^2 & c(\frac{a}{r}+1) \\ 0 & c(\frac{a}{r}+1) & d^2 \end{pmatrix} \frac{m_t^2}{m_R}$$

To make θ_{23} large $\Rightarrow$ $r \sim \frac{ac}{d^2} \sim * \sqrt{\frac{m_u^2 m_c}{m_t^3}} \sim 10^{-(6-8)}$

M_ν is determined!

$$M_\nu \simeq \begin{pmatrix} 0 & \frac{a^2}{r} & 0 \\ \frac{a^2}{r} & \frac{2ab}{r} & \frac{ac}{r} \\ 0 & \frac{ac}{r} & d^2 \end{pmatrix} \frac{m_t^2}{m_R} \equiv \begin{pmatrix} 0 & \beta & 0 \\ \beta & \alpha & h \\ 0 & h & 1 \end{pmatrix} \frac{d^2 m_t^2}{m_R}$$

with $h \equiv \frac{ac}{rd^2}, \quad \alpha \equiv \frac{2ab}{rd^2}, \quad \beta \equiv \frac{a^2}{rd^2}$

Taking the experimental values

$$\sin^2 2\theta_{23},\ \tan^2\theta_{12},\ \Delta m^2_{32},\ \Delta m^2_{21}$$

$$\overline{M}_\nu(M_R) = \begin{pmatrix} 0 & \beta & 0 \\ \beta & e^{i\phi}\alpha & h \\ 0 & h & 1 \end{pmatrix} \frac{d^2 m_t^2}{m_R}$$

We can obtain the allowed region of

$$\alpha, \beta, h, \phi, \rho, \sigma, m_R$$

In conclusion, we find the best Texture

$$M_U, M_{\nu_D};\ \begin{pmatrix} 0 & 126 & 0 \\ 126 & 10 & 10 \\ 0 & 10 & 126 \end{pmatrix}$$

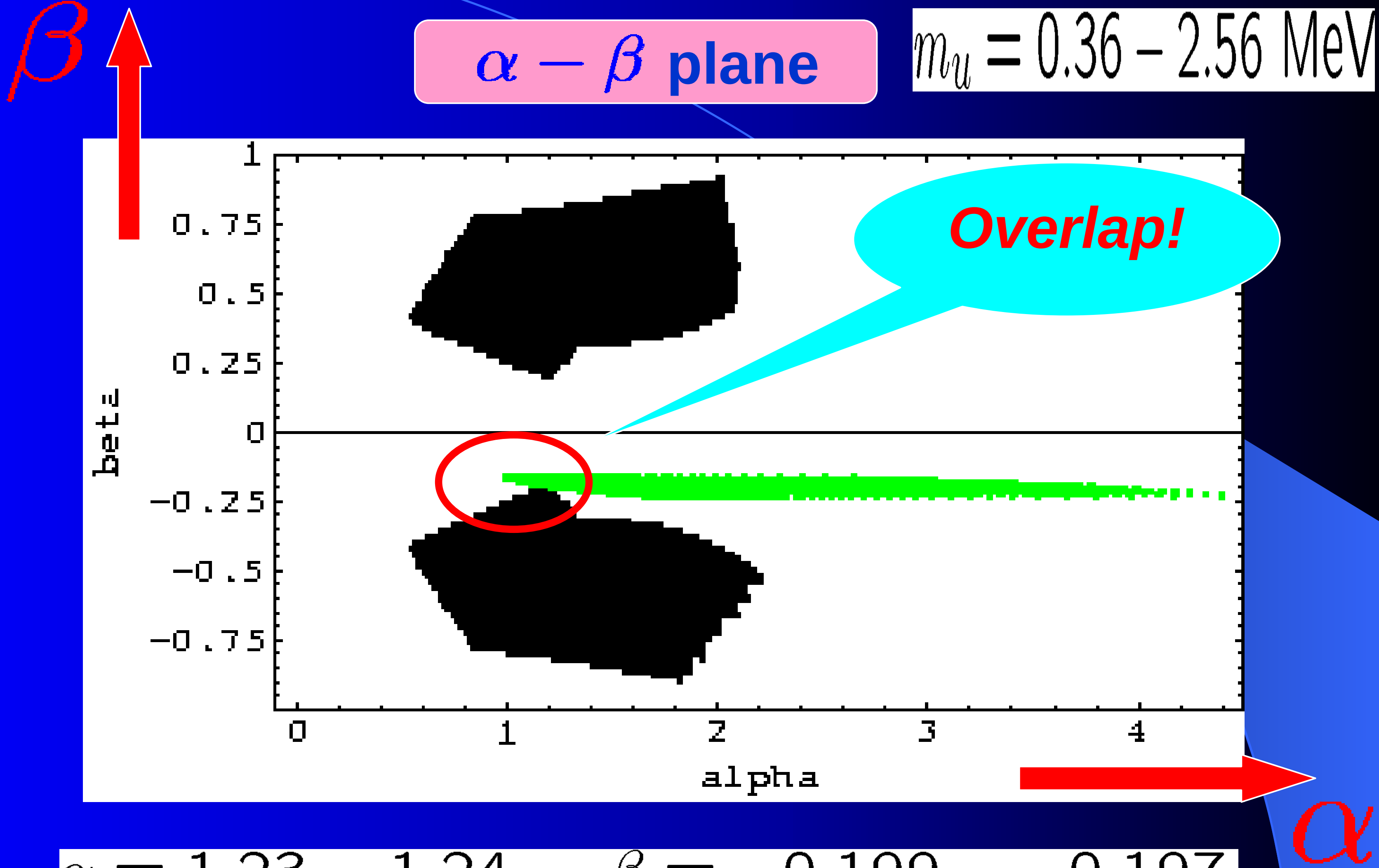
for

$$M_R = m_R \begin{pmatrix} 0 & r & 0 \\ r & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In the case of $h = 1.3$

$\alpha - \beta$ **plane**

$m_u = 0.36 - 2.56$ MeV



$$\alpha = 1.23 \sim 1.24, \quad \beta = -0.199 \sim -0.197,$$

$$\phi = -\pi/18 \sim \pi/18, \quad \rho = 7\pi/9 \sim 11\pi/9.$$

In the case of $h = 1.3,\ m_R = 3.0 \times 10^{15}$ GeV

$$\sin^2 2\theta_{23} \sim 0.98, \quad \tan^2\theta_{12} \sim 0.28$$

We can obtain the predicted values as follows:

$$\begin{aligned}
|U_{e3}| &= 0.010 \sim 0.048, \\
|J_{CP}| &\leq 9.6 \times 10^{-3}, \\
|m_{\nu_3}| &\sim 0.062\text{eV}, \\
|m_{\nu_2}| &\sim 0.0075\text{eV}, \\
|m_{\nu_1}| &\sim 0.0014\text{eV}, \\
|< m_{ee} >| &\simeq 0.0027\text{eV}.
\end{aligned}$$

$$M_{R1} = 1.0 \times 10^9 \text{ GeV}$$

Texture Zero reproduces Bi-Large Mixing

Bi-Large Mixing is given See-saw,
Seesaw Enhancement

These texture of m_D is consistent with

$\mu \rightarrow e\gamma$ suppression (Left-handed)

Thermal Leptogenesis (Right-handed)

SO(10) GUT approach