

**Lepton Flavor Violation**  
**in**  
**Long-Baseline Experiments**

**Sato , Joe**  
**(Saitama U)**

# 1 Introduction

- Solar neutrino, Atmospheric neutrino, Reactor neutrino

  - ◇ Massive Neutrinos

  - ◇ Lepton Mixing

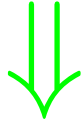
- Massive Neutrino

  - Massive but Very Tiny  $\implies$  Seesaw Mechanism and/or  $\dots$

- Lepton Mixing

  - Large Mixing  $\implies$  Interactions with Large Lepton Flavor Violation (LFV)

and/or  $\dots$



Large Lepton Flavor Violating Process in Our World !?

- Yes, MSSM with Seesaw ( $\nu_R$ ) Borzumati and Masiero, Hisano *et. al.*

Large Flavor Changing Slepton Mass thorough renormalization

even if universal scalar mass( $m_0^2$ ) at GUT scale( $M_G$ )

(Dirac) Neutrino Yukawa couplings

$$W = f_\nu^{ij} \bar{N}_i L_j H_u$$

$$\begin{aligned} \mu \frac{d(m_{\tilde{L}}^2)_{ij}}{d\mu} &= \left( \mu \frac{d(m_{\tilde{L}}^2)_{ij}}{d\mu} \right)_{\text{MSSM}} (= 0) \\ &+ \frac{1}{16\pi^2} \left[ m_{\tilde{L}}^2 f_\nu^\dagger f_\nu + f_\nu^\dagger f_\nu m_{\tilde{L}}^2 + 2(f_\nu^\dagger m_{\tilde{\nu}}^2 f_\nu + \tilde{m}_{H_u}^2 f_\nu^\dagger f_\nu + A_\nu^\dagger A_\nu) \right]_{ij} \end{aligned}$$

SUSY breaking  $m_{\tilde{L}}^2$  scalar lepton doublet

$m_{\tilde{\nu}}^2$  right-handed sneutrino

$\tilde{m}_{H_u}^2$  doublet Higgs

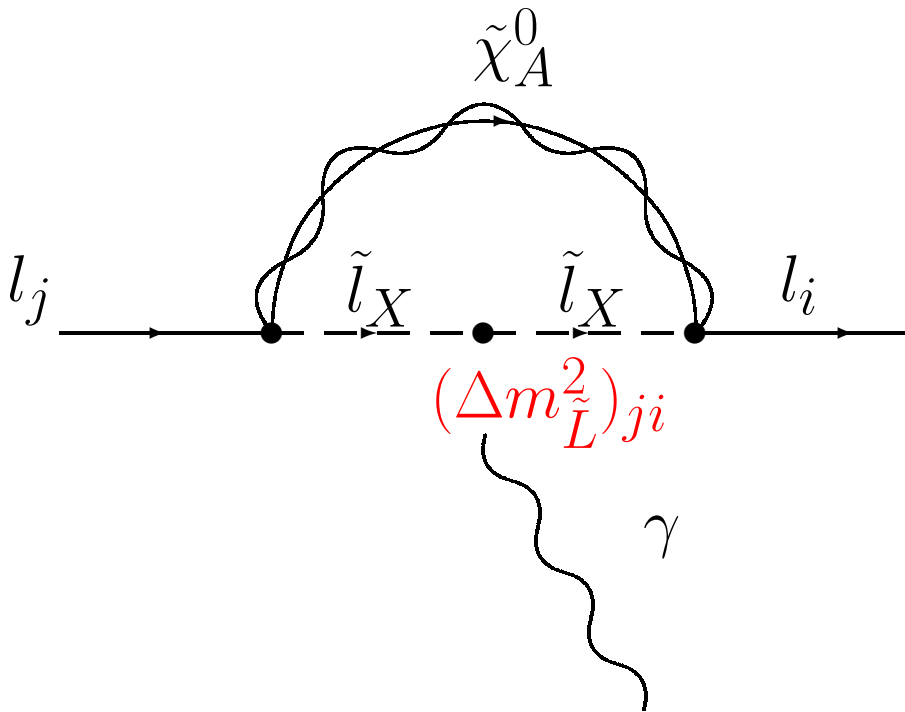
$$U^{Dirac T} f_\nu^{ij} V^{Dirac*} = \text{diag}(f_{\nu 1}, f_{\nu 2}, f_{\nu 3})$$

Approximately ( $a_0$ : universal A term)

$$\begin{aligned}
 (\Delta m_{\tilde{L}}^2)_{ij} &\simeq -\frac{(6+a_0^2)m_0^2}{16\pi^2}(f_\nu^\dagger f_\nu)_{ij} \log \frac{M_G}{M_R} \\
 &\simeq -\frac{(6+a_0^2)m_0^2}{16\pi^2} U_{ik}^{Dirac} U_{jk}^{Dirac*} |f_{\nu k}|^2 \log \frac{M_G}{M_R}
 \end{aligned}$$



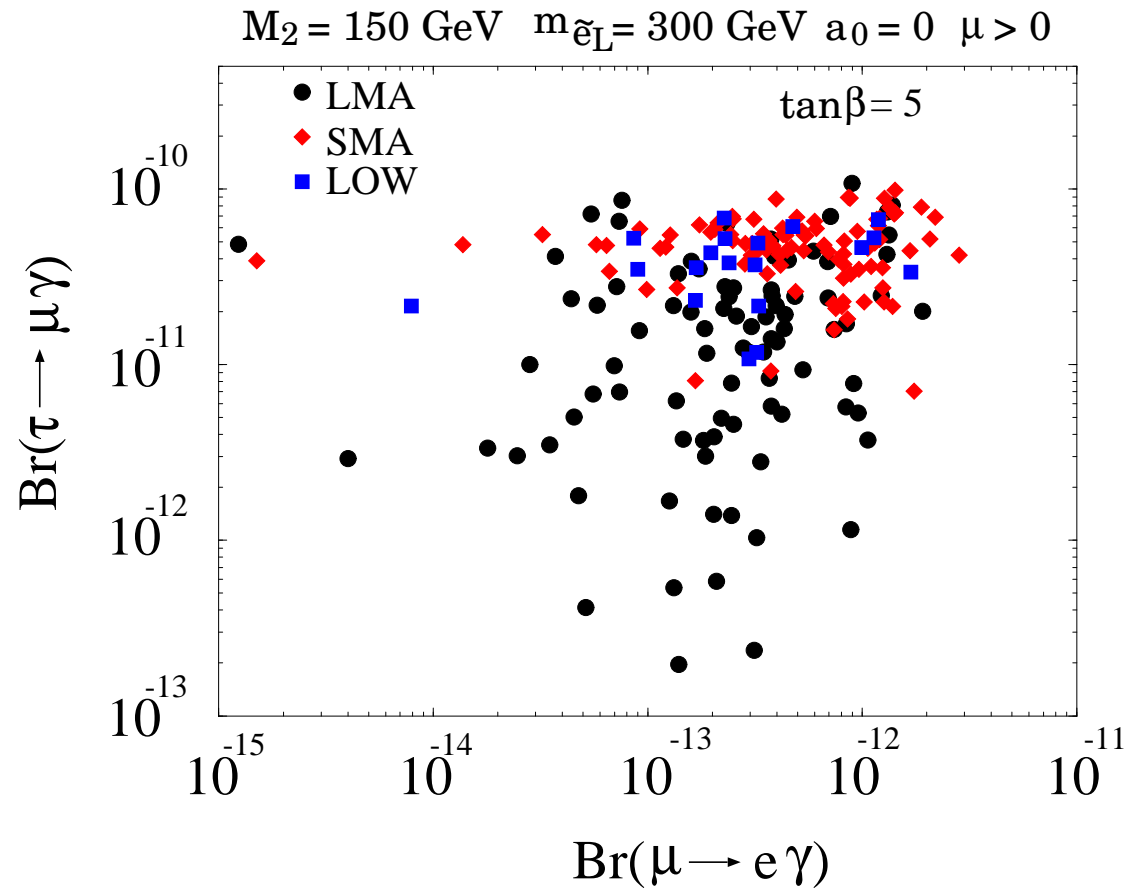
LFV in the charged-lepton



$$\begin{aligned}
 \tau \rightarrow \mu \gamma &\Leftrightarrow (\Delta m_{\tilde{L}}^2)_{32} \\
 \mu \rightarrow e \gamma &\Leftrightarrow (\Delta m_{\tilde{L}}^2)_{21}
 \end{aligned}$$

# Example of Branching Ratio

J.S and K. Tobe



In near future

$$\begin{aligned} \text{Br}(\mu \rightarrow e\gamma) &\sim 10^{-14} \text{ PSI} \\ R(\mu \rightarrow e \text{ in Al}) &\sim 10^{-16} \text{ MECO} \\ R(\mu \rightarrow e \text{ in Ti}) &\sim 10^{-18} \text{ PRISM} \end{aligned}$$

## 2. New Physics in Neutrino Oscillation Experiments

### Quest for LFV

- Precision measurement in (near) future (withn three-generation)

JHF-SK !? Nufact !?

|                       |                    |                        |
|-----------------------|--------------------|------------------------|
| $\delta m_{31}^2$     | : Atmospheric      | 3%                     |
| $\sin^2 2\theta_{23}$ | : neutrino anomaly | 1%                     |
| $U_{e3}(\theta_{13})$ | : Last Mixing      | $\sim 0.01$            |
| $\sin \delta$         | : CP Violation     | $\delta \sim 20^\circ$ |

- Neutrino masses  $\implies$  LFV Interactions  
 $\implies$  Observable Effect in Oscillation Experiments ?

We may see the effect of new physics.

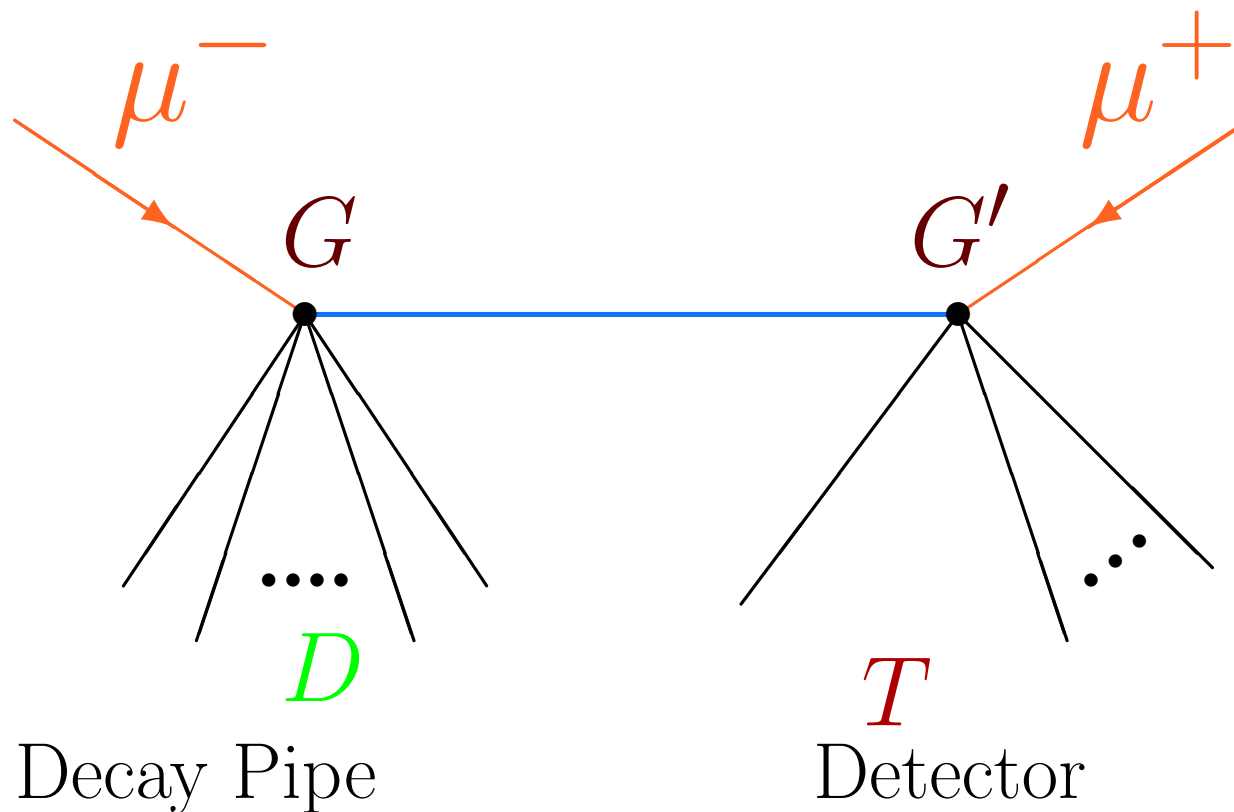
**Gonzalez-Garcia, et. al, Gago, et.al,  
Ota, et al, Huber, et.al, and so on.**

## 2.1 Interference between

Oscillation Amplitude  $\mathcal{A}$  and New-Physics Amplitude  $\mathcal{E}$

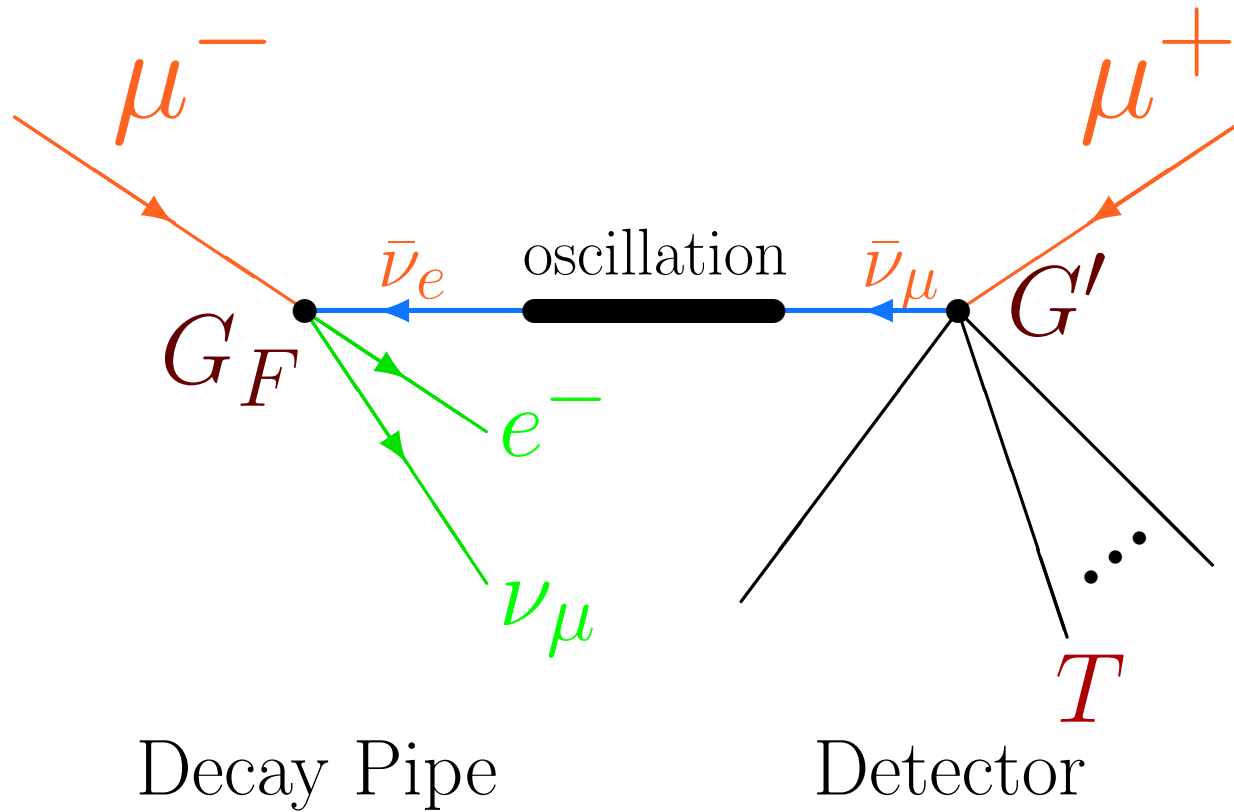
- What we really measure? e.g. in Neutrino Factories

Muons decay and Wrong sign muons appear



- We know there is a weak interaction::

Wrong sign muons suggest the neutrino oscillation

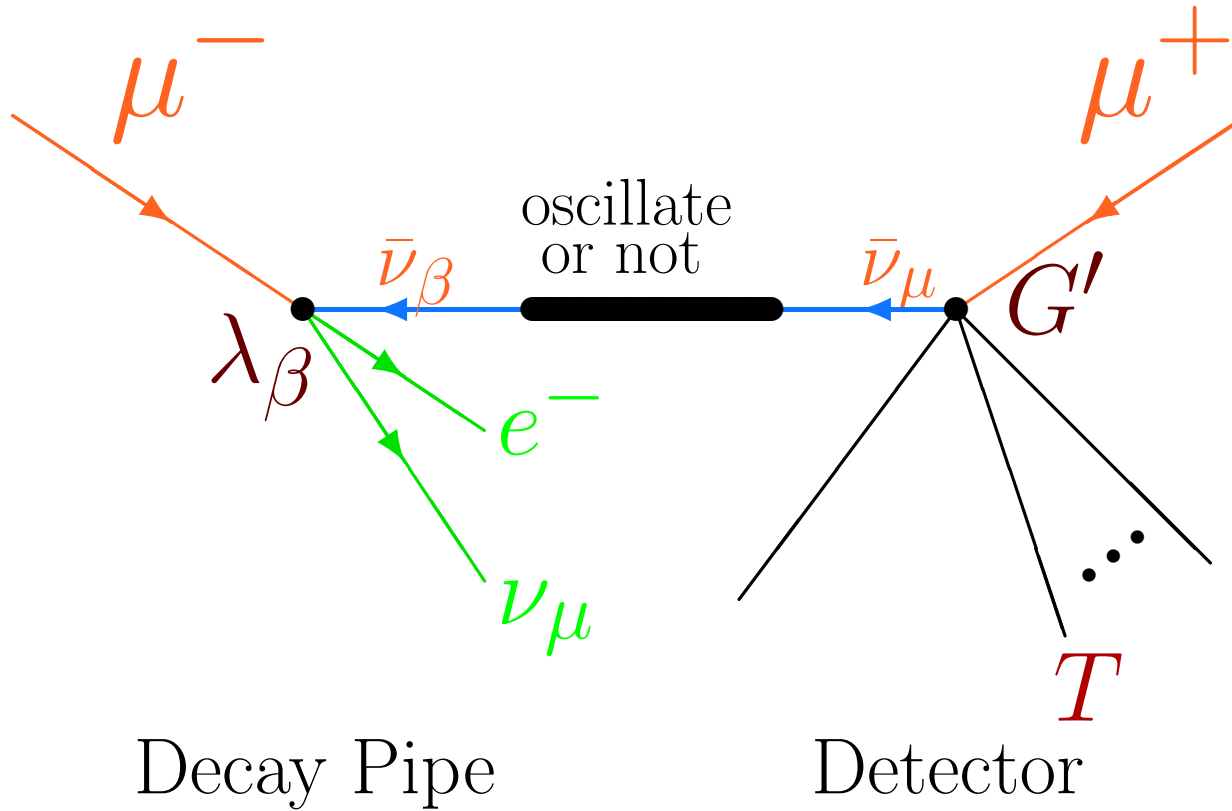




- If there is an interaction,

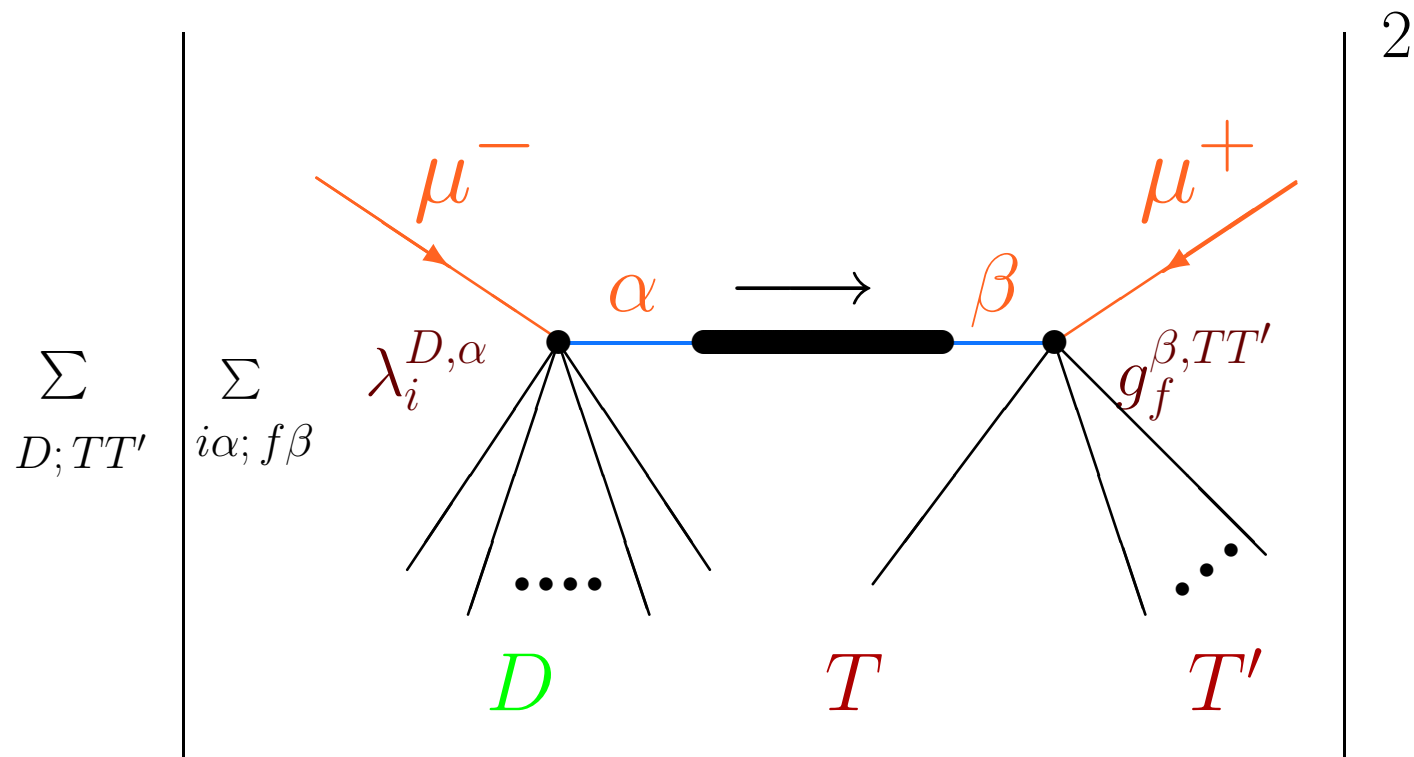
$$L = \lambda(\bar{e}\gamma_{\mu}\mu)(\bar{\nu}_{\mu}\gamma^{\mu}\nu_{\beta}),$$

$$\beta \neq e,$$



Same Signal  $\implies$  Interference

Transition rate for " $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$ "

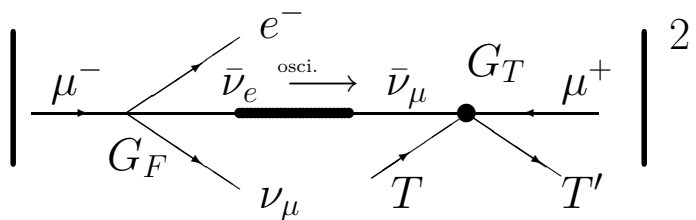


$$\begin{aligned}
& \left| \begin{array}{c} \mu^- \\ \downarrow \\ \text{---} \mu^- \xrightarrow{\bar{\nu}_e \text{ osci.}} \bar{\nu}_\mu \xrightarrow{G_T} \mu^+ \\ \uparrow \\ \nu_\mu \\ \downarrow \\ G_F \\ \uparrow \\ e^- \end{array} \right. + \sum_a \sum_{\alpha=e,\mu,\tau} \left| \begin{array}{c} \mu^- \\ \downarrow \\ \text{---} \mu^- \xrightarrow{\bar{\nu}_\alpha \text{ osci.}} \bar{\nu}_\mu \xrightarrow{G_T} \mu^+ \\ \uparrow \\ \nu_\mu \\ \downarrow \\ \lambda_\alpha^a \\ \uparrow \\ e^- \end{array} \right. \\
& + \sum_b \sum_{\beta=e,\mu,\tau} \left| \begin{array}{c} \mu^- \\ \downarrow \\ \text{---} \mu^- \xrightarrow{\bar{\nu}_e \text{ osci.}} \bar{\nu}_\alpha \xrightarrow{g_\beta^b} \mu^+ \\ \uparrow \\ \nu_\mu \\ \downarrow \\ G_F \\ \uparrow \\ e^- \end{array} \right. + \sum_{a,b} \sum_{\alpha,\beta=e,\mu,\tau} \left| \begin{array}{c} \mu^- \\ \downarrow \\ \text{---} \mu^- \xrightarrow{\bar{\nu}_\alpha \text{ osci.}} \bar{\nu}_\beta \xrightarrow{g_\beta^b} \mu^+ \\ \uparrow \\ \nu_\mu \\ \downarrow \\ T \\ \uparrow \\ e^- \end{array} \right. \Big|^2
\end{aligned}$$

Interference

$$\left| \begin{array}{c} \mu^- \\ \downarrow \\ \text{---} \mu^- \xrightarrow{\bar{\nu}_e \text{ osci.}} \bar{\nu}_\mu \xrightarrow{G_T} \mu^+ \\ \uparrow \\ \nu_e \\ \downarrow \\ \lambda'_e \\ \uparrow \\ e^- \end{array} \right|^2 + \dots + \left| \begin{array}{c} \mu^- \\ \downarrow \\ \text{---} \mu^- \xrightarrow{\bar{\nu}_e \text{ osci.}} \bar{\nu}_\mu \xrightarrow{G_T} \mu^+ \\ \uparrow \\ \nu_\tau \\ \downarrow \\ \lambda'_\tau \\ \uparrow \\ e^- \end{array} \right|^2 + \dots$$

No Interference



and propagation term

$$+ 2 \operatorname{Re} \left[ \left( \mu^- \begin{array}{c} \nearrow e^- \\ \leftarrow \bar{\nu}_e \xrightarrow{\text{osci.}} \bar{\nu}_\mu \\ \searrow \nu_\mu \\ \bullet \\ \nearrow T \\ \searrow T' \end{array} G_T \mu^+ \right)^* \left( \sum_b \sum_{\beta=e,\mu,\tau} \mu^- \begin{array}{c} \nearrow e^- \\ \leftarrow \bar{\nu}_e \xrightarrow{\text{osci.}} \bar{\nu}_\beta \\ \searrow \nu_\mu \\ \bullet \\ \nearrow T \\ \searrow T' \end{array} G_T \mu^+ \right) \right]$$

Interference  $\sim O(\lambda) ::$  linear dependence

$$+ 2 \operatorname{Re} \left[ \left( \mu^- \begin{array}{c} \nearrow e^- \\ \leftarrow \bar{\nu}_e \xrightarrow{\text{osci.}} \bar{\nu}_\mu \\ \searrow \nu_\mu \\ \bullet \\ \nearrow T \\ \searrow T' \end{array} G_T \mu^+ \right)^* \left( \sum_b \sum_{\beta=e,\mu,\tau} \mu^- \begin{array}{c} \nearrow e^- \\ \leftarrow \bar{\nu}_e \xrightarrow{\text{osci.}} \bar{\nu}_\beta \\ \searrow \nu_\mu \\ \bullet \\ \nearrow T \\ \searrow T' \end{array} G_F g_\beta^b \mu^+ \right) \right]$$

Interference  $\sim O(g) ::$  linear dependence

## 2.2 Parametrization

○ New Physics in Decay  $\implies$  Initial State = Flavor Mixed State

If the type of interaction is same

e.g., new physics in muon decay

$$L = \lambda(\bar{e}_L \gamma_\mu \mu_L)(\bar{\nu}_\mu \gamma^\mu \nu_\mu),$$

Initial flavor state:

Grossman

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \implies \begin{pmatrix} 1 \\ \epsilon_{e\mu}^s \\ 0 \end{pmatrix}, \quad \epsilon_{e\mu}^s = \lambda/G_F$$

Pure Electron State  $\implies$  Mixed with Muon State

Otherwise,

T.Ota, J.S, and N. Yamashita

Complicated Energy Dependence and Extra Suppression Factor

$\implies$  Very Probably, No Need to Take into account

○ New Physics in Matter  $\implies$  Shift of Matter Effect

Gago, *et al* Huber, *et al*

$$H \quad + \quad = \quad \begin{pmatrix} & \epsilon_{e\mu}^m & \epsilon_{e\tau}^m \\ \epsilon_{e\mu}^{m*} & \epsilon_{\mu\mu}^m & \epsilon_{\mu\tau}^m \\ \epsilon_{e\tau}^{m*} & \epsilon_{\mu\tau}^{m*} & \epsilon_{\tau\tau}^m \end{pmatrix}.$$

○ New Physics in Detection Process

T. Ota, J.S, and N. Yanashita

Exactly, Need to Consider Parton Distribution

However Simple Treatment Could be done, *i.e.* Flavor Mixed State

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \implies \begin{pmatrix} 1 \\ \epsilon_{e\mu}^d \\ 0 \end{pmatrix}$$

○  $\epsilon$ 's of  $O(10^{-4})$  Can be reach

Gonzalez-Garcia, *et. al*, Gago, *et.al*, Ota, *et al*, Huber, *et.al*,

- Model Independent Constraint on  $\epsilon^{\prime}S$

From SU(2) Inverted Process

Bergman and Grossmann

e.g.  $\mu^- \rightarrow e^- \nu_\tau \bar{\nu}_e \Leftrightarrow \tau^- \rightarrow \mu^- e^- e^+$

$$\implies \epsilon_{\mu\tau}^s \leq 3 \times 10^{-3}$$

factor 2-3 (T. Ota, J.S, N.Yamashita) or maybe 10 (P. Huber and J.W.Valle)

could be multiplied since SU(2) is broken

- Strong Correlation between Some of LFV Effect and Mixing Parameters

$\nu_e \rightarrow \nu_\mu$  channel,  $\epsilon_{e\tau}$  terms in high energy

$$\begin{aligned}
\Delta P_{\nu_e \rightarrow \nu_\mu} \{ \epsilon_{e\tau} \} &= 2s_{23}s_{2 \times 23}s_{2 \times 13} \\
&\times \left[ c_{13}^2 (s_\delta \text{Re}[\epsilon_{e\tau}^s] - c_\delta \text{Im}[\epsilon_{e\tau}^s]) \left( \frac{\bar{a}}{4E_\nu} L \right) \left( \frac{\delta m_{31}^2 L}{4E_\nu} \right)^2 \right. \\
&\quad + c_{13}^2 (c_\delta \text{Re}[\epsilon_{e\tau}^s] + s_\delta \text{Im}[\epsilon_{e\tau}^s]) \\
&\quad \quad \left. \left\{ 1 - \frac{1}{2} \left( \frac{\bar{a}}{4E_\nu} L \right)^2 - s_{13}^2 \left( \frac{\bar{a}}{4E_\nu} L \right) \left( \frac{\delta m_{31}^2 L}{4E_\nu} \right) \right\} \left( \frac{\delta m_{31}^2 L}{4E_\nu} \right)^2 \right. \\
&\quad - c_{13}^2 (s_\delta \text{Re}[\epsilon_{e\tau}^m] + c_\delta \text{Im}[\epsilon_{e\tau}^m]) \left( \frac{\bar{a}}{4E_\nu} L \right) \left( \frac{\delta m_{31}^2 L}{4E_\nu} \right)^2 \\
&\quad - \frac{1}{3} s_{13}^2 (c_\delta \text{Re}[\epsilon_{e\tau}^m] - s_\delta \text{Im}[\epsilon_{e\tau}^m]) \\
&\quad \quad \left. \left. \left\{ \left( \frac{\bar{a}}{4E_\nu} L \right) + 2 \left( \frac{\delta m_{31}^2 L}{4E_\nu} \right) \right\} \left( \frac{\bar{a}}{4E_\nu} L \right) \left( \frac{\delta m_{31}^2 L}{4E_\nu} \right)^2 \right] \right]
\end{aligned}$$

■ All terms are proportional to  $E_\nu^{-2}$  or more lower power terms. This energy dependence is the same as the main oscillation terms.

$$s_{23}^2 s_{2 \times 23}^2 \left( \frac{\delta m_{31}^2 L}{4E_\nu} \right)^2$$



$\nu_e \rightarrow \nu_\mu$  channel,  $\epsilon_{e\mu}$  terms in high energy

$$\begin{aligned}
\Delta P_{\nu_e \rightarrow \nu_\mu} \{ \epsilon_{e\mu} \} &= 2s_{23}s_{2 \times 13} \\
&\times \left[ (s_\delta \text{Re}[\epsilon_{e\mu}^s] - c_\delta \text{Im}[\epsilon_{e\mu}^s]) \left\{ 1 - \frac{2}{3} \left( \frac{\bar{a}}{4E_\nu} L \right)^2 \right\} \left( \frac{\delta m_{31}^2 L}{4E_\nu} \right) \right. \\
&\quad - (c_\delta \text{Re}[\epsilon_{e\mu}^s] + s_\delta \text{Im}[\epsilon_{e\mu}^s]) \left\{ 1 - \frac{1}{3} \left( \frac{\bar{a}}{4E_\nu} L \right)^2 \right\} \left( \frac{\bar{a}}{4E_\nu} L \right) \left( \frac{\delta m_{31}^2 L}{4E_\nu} \right) \\
&\quad + 2c_{23}^2 (s_\delta \text{Re}[\epsilon_{e\mu}^m] + c_\delta \text{Im}[\epsilon_{e\mu}^m]) \left( \frac{\bar{a}}{4E_\nu} L \right) \left( \frac{\delta m_{31}^2 L}{4E_\nu} \right)^2 \\
&\quad \left. + 2 (c_\delta \text{Re}[\epsilon_{e\mu}^m] - s_\delta \text{Im}[\epsilon_{e\mu}^m]) \left\{ 1 - \frac{1}{3} \left( \frac{\bar{a}}{4E_\nu} L \right)^2 \right\} \left( \frac{\bar{a}}{4E_\nu} L \right) \left( \frac{\delta m_{31}^2 L}{4E_\nu} \right) \right] \\
&\quad + \mathcal{O}(1/E^2)
\end{aligned}$$

■ Some of them depend on  $E_\nu^{-1}$ . The terms must be robust against the uncertainty of the parameters.

Uncertainty of Mixing Parameter Hide Some of LFV Effect T. Ota, J.S., N.Yamashita

Conversely,

Weak Constraint on LFV Effect may be Obstacle to Determine Mixing Parameters

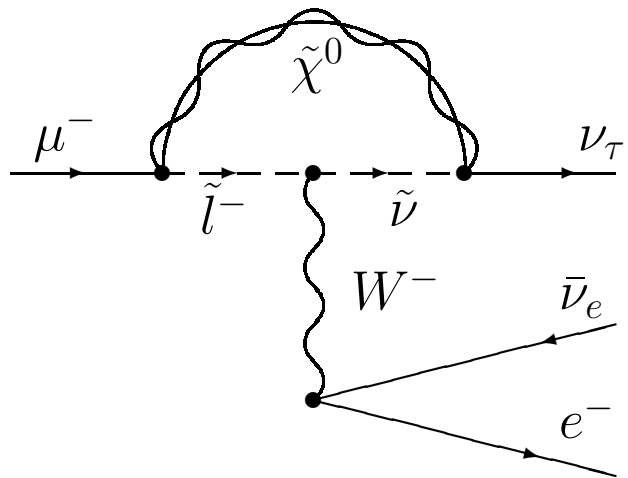
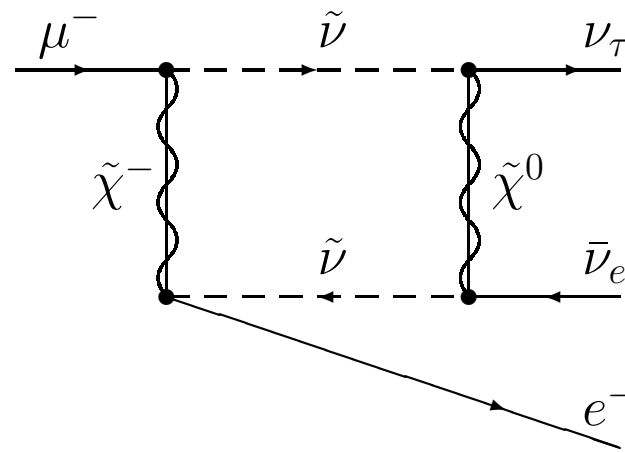
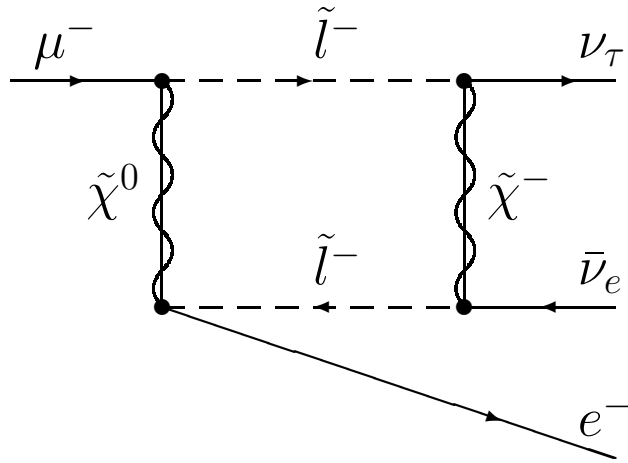
P. Huber and J.W. Valle

Comparison with Different Modes is Important

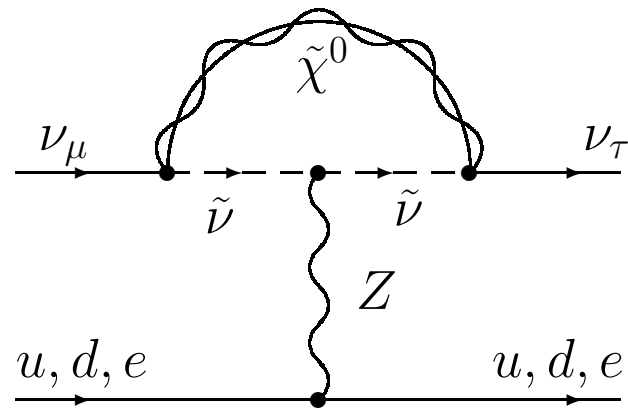
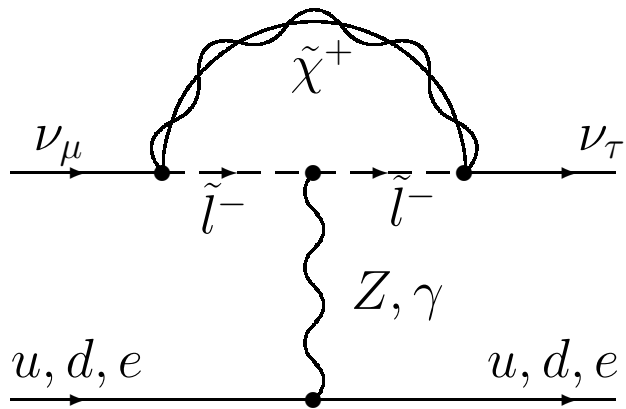
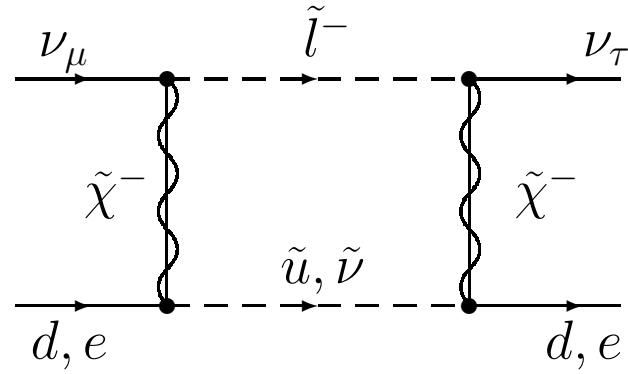
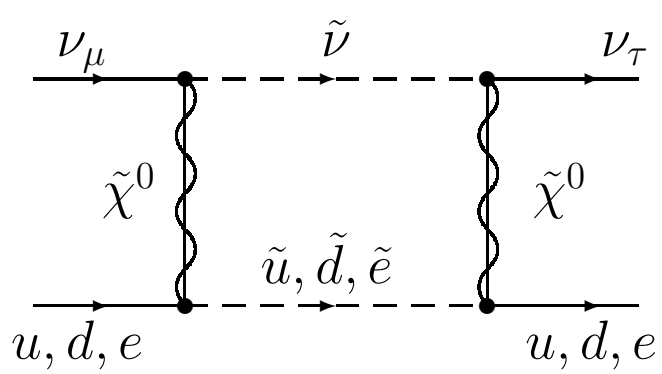
### 3 LFV Effect in MSSM with Right-Handed Neutrinos

○ Similar Effect to  $\mu \rightarrow e\gamma$

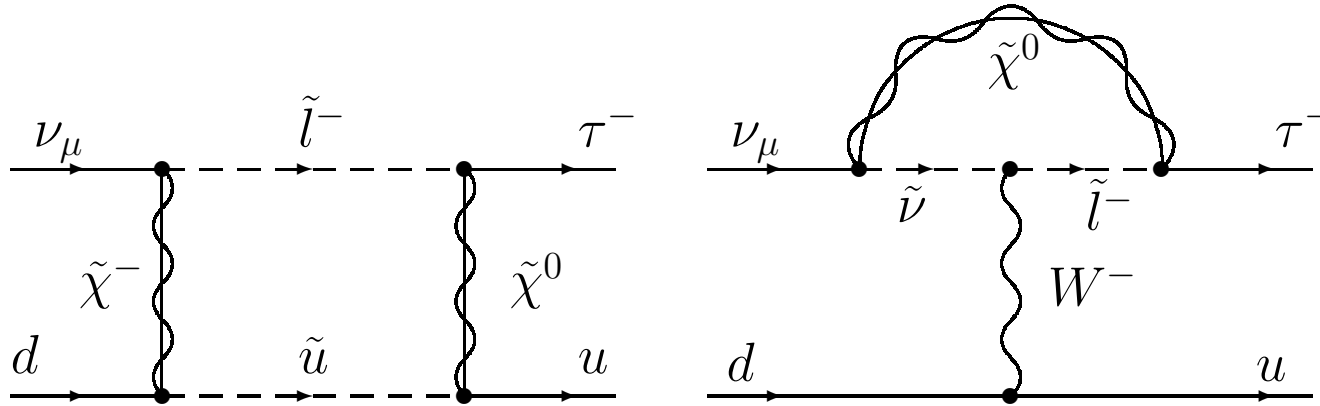
#### Examples of LFV Interaction in Muon Decay



# Examples of LFV Interactions in Matter Effect



# Examples of LFV Interaction in a Detection Process



○ Amplitude of Graphs  $\mathcal{E}$

$$\mathcal{E} \sim \frac{(\Delta m_{\tilde{L}}^2)_{\alpha\beta}}{16\pi^2 m_{\tilde{S}}^4} g^4 \simeq -\frac{(6 + a_0^2)}{16\pi^2} (f_\nu^\dagger f_\nu)_{\alpha\beta} \log \frac{M_G}{M_R} \frac{g^4}{16\pi^2 m_{\tilde{S}}^2}$$

and hence

$$\epsilon = \frac{\mathcal{E}}{G_F} \sim -\frac{(6 + a_0^2)}{16\pi^2} (f_\nu^\dagger f_\nu)_{\alpha\beta} \log \frac{M_G}{M_R} \frac{g^4}{16\pi^2 m_S^2 G_F} \sim -\frac{(6 + a_0^2)}{16\pi^2} (f_\nu^\dagger f_\nu)_{\alpha\beta} \log \frac{M_G}{M_R} \frac{g^2}{16\pi^2}$$

can be  $O(10^{-4})$ .

Moreover, Diagrams Contribute **Coherently**,

Decay, Propagation, Detection,

Factor 10 (?) Enhancement

## 4 Summary and Discussion

- Explanation for Neutrino Masses and Lepton Mixing  
by **MSSM with RH Neutrino**  
~ **Promising**
  - Large LFV Phenomena in Charged Lepton Expected
  - Similarly, Large LFV Phenomena  $O(10^{-3})$  in **Neutrino Oscillation Experiment**  
**Good News ?**      **Obstacle ?**
- Strong **Correlation** between **Some of LFV Effect** and **Mixing Parameters**

**Uncertainty of Mixing Parameter Hide Some of LFV Effect** T. Ota, J.S., N.Yamashita

Conversely,

**Weak Constraint on LFV Effect may be Obstacle to Determine Mixing Parameters**

P. Huber and J.W. Valle



Comparison between Several Modes,

T.Ota and J.S.

$\nu_\mu \rightarrow \nu_\mu$  (T2K ?) **VS**  $\nu_\mu \rightarrow \nu_\tau$  (OPERA ?)

○ Advantage over Direct Detection

$\mathcal{S}$  : Systematic Error

Direct Detection

$$\epsilon^2 > \mathcal{S} \longrightarrow \epsilon > \sqrt{\mathcal{S}}$$

Oscillation Detection

$$\mathcal{A}\epsilon > \mathcal{S} \longrightarrow \epsilon > \frac{\mathcal{S}}{\mathcal{A}} > \sqrt{\mathcal{S}}$$

$\mathcal{A}^2 > \mathcal{S}$  : Always expected

○ Keep in Mind the Possibility of LFV Interactions